Quantum effects on hybrid type inflationary models

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DESY

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Jürgen Baacke, NK. Phys.Rev.D81:023509,2010

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Introduction

Inflationary models are based on the quantum field theory.
Construction of simplified inflationary models
- during the inflation the quantum fluctuations are neglected;
- after inflation expansion of the universe is neglected.

Neither of these is suitable near the exit of inflation.
Especially in the case of the hybrid model which was precisely constructed to describe this exit.
We will consider the system of coupled scalar fields out-of-equilibrium including the quantum back-reaction in the one-loop approximation and coupling of the fields to gravity.

Introduction

Technical aspects:
- The equation of motion for the classical fields and quantum fluctuations constitute the complex system of integro-differential equations;
- analytical solutions are out of reach;
- numerical implementation requires a suitable scheme;
- the quantum back-reacktion involves divergences, which requires a suitable combination of analytical und numerical methods.

The equations of motion

A generalization of the standard hybrid model:
Lagrangian of N coupled scalar fields in curved space-time

$$\mathcal{L} = \sqrt{-g}\left\{\frac{1}{2}\sum_i g^{\mu\nu}\partial_\mu\Phi_i\partial_\nu\Phi_i - V(\Phi) - W(R,\Phi)\right\}$$

with the potential

$$V(\Phi) = \frac{1}{2}\sum_{i=1}^{N} m_i^2\Phi_i^2 + \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_{ij}\Phi_i^2\Phi_j^2$$

the coupling of the scalar fields to the curvature scalar

$$W(R,\Phi) = \sum_i \frac{\xi_i}{2} R\Phi_i^2$$

The equations of motion

- all fields can acquire time dependent vacuum expectation values ("classical fields")
- via the cross couplings λ_{ij} the fields mix, and so do their quantum fluctuations
- the formalism for handling quantum field theory out of equilibrium is the closed-time-path or Keldysh-Schwinger formalism. Though we do not make explicit reference to it, our equations are based on this formalism. In the one-loop approximation they take a simple, intuitive form.

The equations of motion

The fields are split into the expectation values and the quantum fluctuations around them

$$\Phi_i = \phi_i(t) + \psi_i(t, \boldsymbol{x})$$

The equations of motion for classical fields in conformal time

$$\tilde{\phi}_i'' + \left(m_i^2 + (\xi_i - \xi_n)R\right) a^2 \tilde{\phi}_i + a^{4-n} \sum_j \lambda_{ij} \left[(\phi_j^2 - i\tilde{G}_{jj})\phi_i - 2i\tilde{G}_{ij}\phi_j\right] = 0$$

where

$$\xi_n = \frac{n-2}{4(n-1)}$$

- note that, with hindsight to dimensional regularization we work in $n \neq 4$ dimensions. For $n = 4$ the conformal coupling $\xi_n \to 1/6$;
- the G_{ij} are Green's functions and describe the quantum back-reaction; they will be given below.

The equations of motion for the fluctuations

Expansion of the fluctuation fields in terms of mode functions

$$\tilde{\psi}_i(\tau, \boldsymbol{x}) = \int \frac{d^{n-1}k}{(2\pi)^{n-1}} e^{i\boldsymbol{k}\boldsymbol{x}} f_i(\tau; \boldsymbol{k})$$

The mode equations

$$f_i{}''(\tau; \boldsymbol{k}) + k^2 f_i(\tau; \boldsymbol{k}) + \sum_j \tilde{\mathcal{M}}^2_{ij}(\tau) f_j(\tau; \boldsymbol{k}) = 0$$

The effective masses

$$\begin{aligned}
\tilde{\mathcal{M}}^2_{ii}(\tau) &= \left(m_i^2 + (\xi_i - \xi_n)R\right) a^2 + a^{4-n} \left(3\lambda_{ii}\tilde{\phi}_i{}^2 + \sum_{j \neq i} \lambda_{ij}\tilde{\phi}_j^2\right) \\
\tilde{\mathcal{M}}^2_{ij}(\tau) &= 2a^{4-n}\lambda_{ij}\tilde{\phi}_i\tilde{\phi}_j \qquad i \neq j
\end{aligned}$$

The fluctuation integrals

Quantum back-reaction is mediated by the Green's functions of the quantum fluctuations. These are expressed by the mode functions as

$$G_{ij} = i < \psi_i \psi_j >= i\mathcal{F}_{ij} = \int \frac{d^{n-1}k}{(2\pi)^{n-1}} \frac{1}{2\Omega} f_i(j) f_j(k)$$

where the $\mathcal{F}_{ij}$ will be denoted as "fluctuation integrals"
- the mixing of the fields described by the mass matrix leads to nondiagonal Green' s functions. This has occasionally be "simplified" in the literature on the hybrid model
- the fluctuation integrals are divergent, this will be handled via dimensional regularisation.

The energy-momentum tensor

In the Friedmann equations appear only T_{tt} and T^{μ}_{μ}

$$
\begin{aligned}
T^{\rm cl}_{tt} &= \frac{1}{a^n}\left\{\frac{1}{2}\sum_i(\tilde{\phi}'^2_i + m_i^2 a^2 \tilde{\phi}_i^2) + \frac{a^{4-n}}{4}\sum_{ij}\lambda_{ij}\tilde{\phi}_i^2\tilde{\phi}_j^2 \right. \\
&\left. +2(n-1)\sum_i (\xi_i - \xi_n)\left(aH\tilde{\phi}'_i - \frac{n-2}{4}a^2H^2\tilde{\phi}_i\right)\tilde{\phi}_i\right\} \\
T^{\rm q}_{tt} &= T^{\rm q,kin}_{tt} + \frac{1}{2a^n}\mathcal{V}_{ij}\mathcal{F}_{ij} + \frac{n-1}{a^n}\sum_i (\xi_i - \xi_n)\left[aH\frac{d}{d\tau}\mathcal{F}_{ii}\right. \\
&\left. -\left(\frac{n-2}{2}a^2H^2 + \frac{Ra^2}{2(n-1)}\right)\mathcal{F}_{ii}\right]
\end{aligned}
$$

The energy-momentum tensor

$$T_\mu^{\mathrm{cl}\,\mu} = \frac{1}{a^n}\Bigg\{2(n-1)\sum_i(\xi_i-\xi_n)\left[\tilde{\phi}_i' - \frac{1}{2}(n-2)aH\tilde{\phi}_i\right]^2$$

$$+2(n-1)\sum_i \xi_i\tilde{\phi}_i\tilde{\phi}_i''$$

$$+n\left[\frac{1}{2}\sum_i m_i^2\tilde{\phi}_i^2 + \frac{1}{4}\sum_{ij}\lambda_{ij}a^{4-n}\tilde{\phi}_i^2\tilde{\phi}_j^2 + \Lambda\right]\Bigg\}$$

$$T_\mu^{\mathrm{q}\,\mu} = \frac{n-1}{a^n}\sum_i(\xi_i-\xi_n)\left[\frac{d^2}{d\tau^2}\mathcal{F}_{ii} - (n-2)aH\frac{d}{d\tau}\mathcal{F}_{ii}\right.$$

$$\left. +\frac{1}{2}\left((n-2)^2H^2 - \frac{n}{n-1}R\right)\mathcal{F}_{ii}\right] + \frac{1}{a^n}\sum_{ij}\mathcal{M}_{ij}^2\mathcal{F}_{ij}$$

The counterterm Lagrangian

$$\mathcal{L}^{\mathrm{ct}} = -\frac{1}{2}\left(\delta m_i^2 + \delta\xi_i R\right) a^2 \tilde{\phi}_i^2 - \frac{a^{4-n}}{4}\delta\lambda_{ij}\tilde{\phi}_i^2\tilde{\phi}_j^2 + a^n\left(\frac{1}{2}\delta\tilde{Z}R + \frac{1}{2}\delta\tilde{\alpha}R^2 - \delta\tilde{\Lambda}\right)$$

In dimensional regularisation we find the standard counterterms for the Minkowski-space quantum field theory

$$\delta m_i^2 = \frac{1}{16\pi^2}L_\epsilon[3\lambda_{ii}m_i^2 + \sum_{j\neq i}\lambda_{ij}m_j^2]$$

$$\delta\lambda_{ii} = \frac{1}{16\pi^2}L_\epsilon[9\lambda_{ii}^2 + \sum_{j\neq i}\lambda_{ij}^2]$$

$$\delta\lambda_{ij} = \frac{1}{16\pi^2}L_\epsilon\lambda_{ij}\left[3\lambda_{ii} + 3\lambda_{jj} + 4\lambda_{ij}\right]$$

The counterterm Lagrangian

and the counter terms for the couplings to gravity

$$\delta\tilde{\Lambda} = \frac{1}{64\pi^2} L_\epsilon \sum_i m_i^4$$

$$\delta\xi_i = \frac{1}{16\pi^2} L_\epsilon [3\lambda_{ii}(\xi_i - 1/6) + \sum_{j\neq i} \lambda_{ij}(\xi_j - 1/6)]$$

$$\delta\tilde{Z} = -\frac{1}{16\pi^2} L_\epsilon \sum_i (\xi_i - 1/6) m_i^2$$

$$\delta\tilde{\alpha} = -\frac{1}{32\pi^2} L_\epsilon \sum_i (\xi_i - 1/6)^2$$

- the latter terms vanish for conformal coupling $\xi_i = 1/6$

Finite equations of motion

The fluctuation integrals can be split

$$\mathcal{F}_{ij} = -\frac{L_\epsilon}{16\pi^2}\tilde{\mathcal{M}}^2_{ij}(\tau) + \mathcal{F}^{\rm fin}_{ij} + \mathcal{F}^{\rm add}_{ij}$$

the finite part

$$\mathcal{F}^{\rm fin}_{ij} = \mathcal{F}^{\rm sub}_{ij} + \mathcal{F}^{\rm ft}_{ij}$$

- $\mathcal{F}^{\rm add}_{ij}$ arises from the expansion of $\tilde{\mathcal{M}}^2_{ii}$ for $n \neq 4$.
- the subtracted parts $\mathcal{F}^{\rm sub}_{ij}$ are the integrals over the mode functions, with the leading parts in powers of momentum subtracted in the integrand. They are finite and are computed using the numerical solutions of the mode equations.

The renormalised equations of motion for the classical field

$$\tilde{\phi}''_i + (m_i^2 + (\xi_i - \frac{1}{6})R)a^2\tilde{\phi}_i + \sum_j \lambda_{ij}\left[(\tilde{\phi}_j^2 + \mathcal{F}^{\rm fin}_{jj} + \mathcal{F}^{\rm add}_{jj})\tilde{\phi}_i + 2(\mathcal{F}^{\rm fin}_{ij} + \mathcal{F}^{\rm add}_{ij})\tilde{\phi}_j\right] = 0$$

Simulation: hybrid model

We present the results of simulations for the system of only two fields.

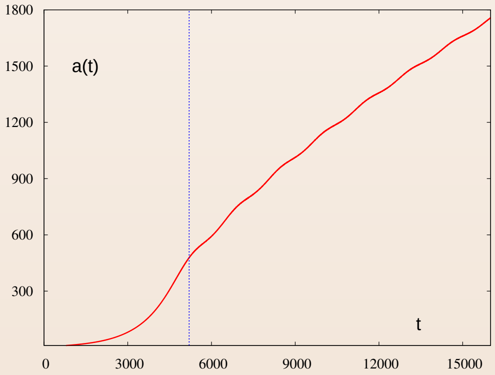
$$\mathcal{L} = \sqrt{-g}\left\{\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\alpha(\chi^2 - v^2)^2 - \frac{1}{2}\lambda\phi^2\chi^2 - \frac{1}{12}R\left(\phi^2 + \chi^2\right)\right\}$$

Simulations are restricted to the case of conformal coupling
($\xi_1 = \xi_2 = 1/6$).
The parameter set (in Planck units):

$$\begin{aligned} v &= 1 \\ m &= 0.01 \\ \lambda &= 10^{-5} \\ \alpha &= 10^{-5} \end{aligned}$$

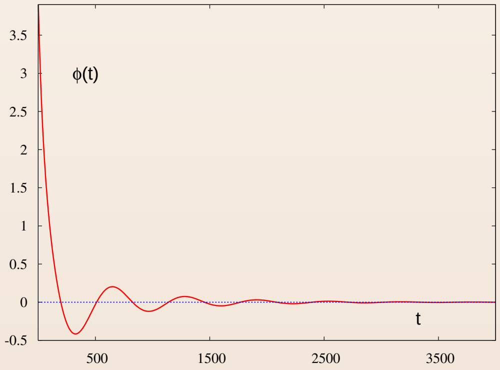
Simulation: hybrid model

Evolution of the scale parameter



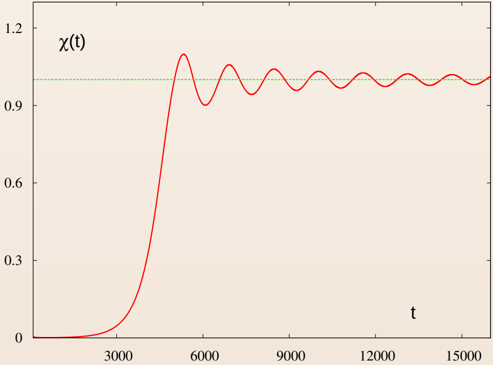
Simulation: hybrid model

Evolution of the inflaton field



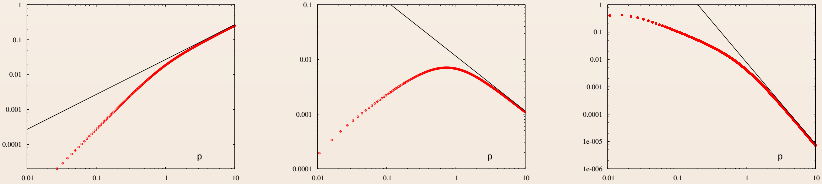
Simulation: hybrid model

Evolution of the waterfall field



Simulation: hybrid model

Integrand of the fluctuation integral:



without subtraction

after removing the leading order

fully subtracted

Outlook

- We have presented the general analysis of the renormalisation of a set of coupled scalar fields in FRW universe with non-minimal gravitational coupling in one-loop approximation;
- by a suitable subtraction procedure the equations of motion and the energy-momentum tensor become finite and numerically well-behaved;
- the renormalisation procedure was applied to the hybrid model of inflation;
- renormalisation procedure is solid and gives very stable numerical results;
- we expect it to be easily generalised to contain fermionic fields;
- realistical and phenomenological application.