Smeared versus localised sources in flux compactifications

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Overview

1. Introduction
2. BPS solutions with Ricci-flat internal space
3. BPS solutions with negatively curved twisted tori
4. Non-BPS solutions
5. Discussion

Introduction: What is the issue?

Consider classical vacuum solutions to type II supergravity with Dp -branes ($\mu_p < 0$) or Op -planes ($\mu_p > 0$):

$$S_{\rm loc} = \mu_p \mathrm{e}^{\frac{p-3}{4}\phi} \int \mathrm{d}^{10}x \sqrt{|g|}\delta^{(9-p)}(x) - \mu_p \int C_{p+1} \wedge \delta^{(9-p)}$$

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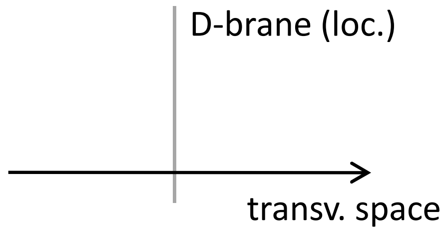
Popular idea: simplify computations by assuming

$$\boxed{\delta^{(9-p)}(x) \to \text{const.}}$$

"smearing"



But: D-branes/O-planes are localised objects
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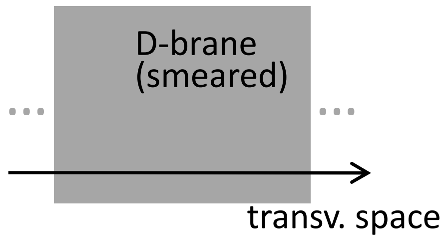

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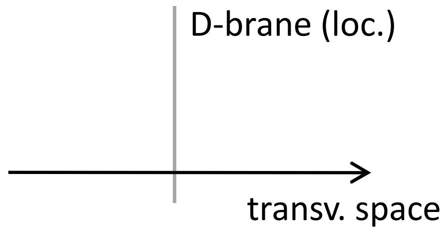

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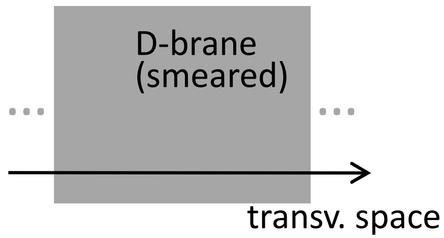

→ Smearing justified?

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- spaces with negative internal curvature can give uplifting potential
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→ need to understand localisation effects on internal curvature!

BPS solutions (1): smeared limit

Ansatz:

- compactifications down to $p+1$ dimensions with spacetime-filling Op-plane ($1 \leq p \leq 6$)

[S. Giddings, S. Kachru, J. Polchinski 2001]

- generalisation of well-known GKP solution, related to it by T-duality
- non-zero fields: $\phi = \phi_0, \quad H, \quad F_{6-p}$
- metric: $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{ij}\mathrm{d}x^i \mathrm{d}x^j, \quad R_{\mu\nu} = 0$

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All e.o.m. (Einstein, dilaton, field strengths, Bianchi id's) solved with conditions

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→ $(p+1)$-dimensional Minkowski solutions with Ricci-flat internal space

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$$F_{8-p} = (-1)^p\,\frac{16}{7-p}\mathrm{e}^{\frac{16}{p-7}A - \frac{p-3}{4}\phi_0}\tilde{\star}_{9-p}\mathrm{d}A,$$

$$\mu_p\tilde{\delta}(\mathrm{O}p) = \mathrm{e}^{-\frac{p+1}{4}\phi_0}|\tilde{H}|^2 + \mathrm{e}^{-\frac{p-3}{4}\phi_0}\tilde{\nabla}^2\mathrm{e}^{\frac{16}{p-7}A},$$

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→ solutions can be localised, if we add warping, a varying dilaton and F_{8-p}

BPS solutions (2): smeared limit

Ansatz:

- compactifications down to p dimensions with Op-plane filling spacetime and non-closed internal direction e^9
- generalisation of [S. Kachru, M. Schulz, P. Tripathy, S. Trivedi 2003] , related to GKP by T-dualities
- non-zero fields: $\phi = \phi_0, \quad F_{8-p} = m_{7-p} \wedge e^9$
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$\mathrm{d}e^9 = \frac{1}{2} f^9_{ij} e^i \wedge e^j$

not closed! Hence find:

$R_{99} = \frac{1}{2} g_{99} g_{99} |\mathrm{d}e^9|^2 + \tilde{R}_{99}$

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→ p-dimensional Minkowski solutions with negatively curved twisted tori

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Non-BPS solutions

Now consider again the same ansatz as above, i.e.

- non-zero fields: $\phi, \quad H, \quad F_{6-p}, \quad F_{8-p}$
- some warped metric: $\mathrm{d}s^2 = \mathrm{e}^{2A}\mathrm{d}\tilde{s}^2_{p+1} + \mathrm{d}s^2_{9-p}$

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Check a simple example ($p = 3, \kappa = -1$) to see what happens...

Go again through the e.o.m. to find:

$\mu_3 < 0 \qquad \rightarrow$ D3-brane

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BPS ($\kappa = 1$):

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→ smeared solution gives AdS, but we cannot make sense out of localisation

Discussion

When does a smeared solution imply a localised solution?

- possibility of promoting smeared solutions to localised ones appears to rely on whether solutions are BPS or not
- for more complicated examples (e.g. with intersecting sources), localisation may be more involved or even impossible

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What about localisation effects on the internal curvature?

- [M. Douglas, R. Kallosh 2010] argued that uplifting potentials by means of localized sources are problematic
- localising our solutions keeps integrated internal scalar curvature negative

→ localisation effects seem to be more involved than expected before

Thank you!