Wrapped branes, consistent truncations and AdS/CMT

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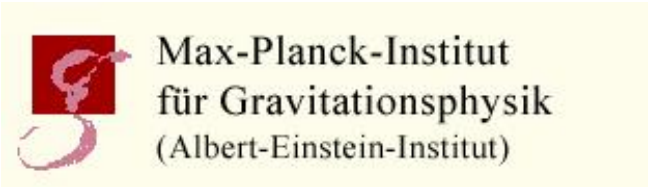

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Quantum Field Theory: Developments and Perspectives

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Outline

Bottom-up AdS/CMT

- Strongly coupled field theories can be studied via AdS/CFT.

  Condensed matter applications.

- Holographic superconductors can be studied in the Einstein-Maxwell-charged scalar-AdS system [Gubser '09; Hartnoll, Herzog, Horowitz '08]

$$e^{-1}\mathcal{L} = R + 2\Lambda - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - D_\mu\phi D^\mu\phi^* - V(\phi,\phi^*), \quad D\phi = d\phi - iqA_1\phi$$

  by studying perturbations to AdS-RN-like black holes.

- Non-relativistic, scale-invariant theories can be holographically described by solutions to the Einstein-Proca-AdS model [Son '08; Balasubramanian, McGreevy '08]

$$e^{-1}\mathcal{L} = R + 2\Lambda - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}m^2A_\mu A^\mu$$

Top-down AdS/CMT

For a precise holographic dictionary to exist, these models should be embedded into string theory.

Several available techniques:

- D-brane engineering [Ammon, Erdmenger, Kaminski, Kerner '09, ...]
- Duality transformations [Herzog, Rangamani, Ross '08; Maldacena, Martelli, Tachikawa, '08; Adams, Balasubramanian, McGreevy '08]
- Consitent Kaluza-Klein truncations [Buchel, Liu '06; Maldacena, Martelli, Tachikawa, '08; Gauntlett, Kim, OV, Waldram '09; Gauntlett, Sonner, Wiseman '09; ...]

Consistent KK truncations

- A KK truncation of D-dimensional (super)gravity reduced on M_{D-d} is consistent if the retained modes do not source the discarded modes.
- Eg: $D = 5$ gravity on S^1 consistently truncates to $D = 4$ gravity $+\phi + A_\mu$.
- The retained modes define a d-dimensional theory in their own right.
- d-dim solutions uplift on M to D-dim solutions.

Conventional lore:

- Consistent truncations are very rare.
- Truncations containing a finite number of massive modes should be even even rarer.

Top-down AdS/CMT? Remain skeptic, but open minded.

Massless sector truncations

- The KK tower of $D = 10, 11$ sugra on spheres consistently truncates to the full massless sector, giving the maximal (pure) $N = 8$ gauged supergravities [De Witt, Nicolai; Nastase, Vaman, vanNieuwenhuizen; Cvetic, Pope, Warner...]

- More generally:
  - Theorem. For any supersymmetric $AdS_d \times_w M_{D-d}$ solution of $D = 10, 11$ sugra there is a consistent KK truncation on M_{D-d} to the pure d-dim gauged sugra Gauntlett, OV '07].
  - Proof: G-structure technology.

- For lower susy reductions, the full massless sector involves matter-coupled sugra, and cannot be always retained consistently [Duff, Nilsson, Pope, Warner '84; Hoxha, Martínez-Acosta, Pope '00]

Embedding into string theory

- top-down AdS/CMT needs consistent truncations beyond the massless level.
- The model allowing for non-relativistic duals can be embedded into
  - type IIB supergravity [Herzog, Rangamani, Ross '08; Maldacena, Martelli, Tachikawa '08; Adams, Balasubramanian, McGreevey, '08].
  - $D = 11$ supergravity [Gauntlett, Kim, OV, Waldram '09]
- The holographic superconductor model can be embedded into
  - type IIB supergravity [Gubser, Herzog, Pufu, Tesileanu'09]
  - $D = 11$ supergravity [Gauntlett, Sonner, Wiseman '09]

Planar M2s, near horizon

Place M2 branes on the apex of a special holonomy cone. The near horizon solution is of the form $AdS_4 \times M_7$, where M_7 is an Einstein manifold [Freund, Rubin '80]

$$\begin{aligned} ds^2_{11} &= \tfrac{1}{4} ds^2(AdS_4) + ds^2(M_7) \\ G_4 &= \tfrac{3}{8} \,\mathrm{vol}(AdS_4) \end{aligned}$$

If the cone $(C(M_7), dr^2 + r^2 ds^2(M_7))$ has special holonomy, there exist Killing spinors on M_7 and the $D = 11$ FR solution is supersymmetric. Four cases [Acharya, Figueroa-O'Farrill, Hull, Spence '98] :

susy	M_7	$C(M_7)$	G-structure on M_7
$N = 1$	Weak G_2	$Spin(7)$	φ_3
$N = 2$	Sasaki-Einstein	Calabi-Yau	(η, J, Ω)
$N = 3$	3-Sasaki	Hyper-Kähler	$(\eta^a, J^a), a = 1, 2, 3$
$N = 4, 5, 6, 7, 8$	S^7/Γ	$\mathbb{R}^8$	identity

Planar branes and consistent truncations

[Gauntlett, Kim, OV, Waldram '09]

D, brane	M	$C(M)$	truncated theory
$D = 11$, M2	Weak G_2	$Spin(7)$	$d = 4$ $N = 1$ gauged sugra + 1 CM
$D = 11$, M2	Sasaki-Einstein$_7$	Calabi-Yau$_4$	$d = 4$ $N = 2$ gauged sugra + 1 VM + 1HM
$D = 11$, M2	3-Sasaki$_7$	Hyper-Kähler$_4$	$d = 4$ $N = 4$ gauged sugra + 3 VM

[Cassani, Dall'Agata, Faedo '10] [Gauntlett, OV '10] [Liu, Szepietowski, Zhao '10]

IIB, D3	Sasaki-Einstein$_5$	Calabi-Yau$_3$	$d = 5$ $N = 4$ gauged sugra + 2 VM

Further developments:

- Particularisation to specific Sasaki-Einstein: S^5 [Liu, Pope '10], $T^{1,1}$ [Cassani, Faedo '10] [Bena, Giecold, Grana, Halmagyi, Orsi '10]
- Fermions [Bah, Faraggi, Jottar, Leigh, P.Zayas '10]

Branes wrapping cycles

- Branes can partially wrap calibrated cycles of special holonomy manifolds whilst preserving supersymmetry [Maldacena, Nunez '00]

- Eg: Slag-3 cycle Σ_3 in CY_3:

  $$\mathrm{vol}(\Sigma_3) = (\mathrm{Re}\Omega)\,|_{\Sigma_3}$$

  where Ω is the holomorphic $(3,0)$ form on CY_3.

- M5 branes wrapping Σ_3 are 1/4 BPS. Worldvolume: $\mathbb{R}^{(1,2)} \times \Sigma_3$. R-sym currents active on Σ_3.

- For $\Sigma_3 = H_3$: flow across dimensions to a $d = 3$ $N = 2$ CFT, dual to $AdS_4 \times H^3 \times S^4$.

  [Gauntlett, Kim, Waldram '00]

Branes wrapping cycles: consistent truncation

Associated to M5 branes wrapping a slag Σ_3 in CY_3, there exists a consistent truncation of $D = 11$ sugra on $\Sigma_3 \times S^4$ down to $D = 4$ sugra.

Two step reduction:

- $D = 11$ supergravity on S^4 down to $D = 7$ $SO(5)$-gauged sugra [Nastase, Vaman, van Nieuwenhuizen '99]

- $D = 7$ $SO(5)$-gauged sugra on Σ_3 down to

  $D = 4$ $N = 2$ gauged supergravity (metric, 1 vector) $+1$ vector multiplet (1 vector, 2 scalars) $+2$ hypermultiplets (8 scalars)

  - Scalar manifold: $\frac{SU(1,1)}{U(1)} \times \frac{G_{2(2)}}{SO(4)}$

  - Gauging: $U(1) \times \mathbb{R} \subset G_{2(2)}$.

Vacuum structure

Two extrema of the scalar potential V identified with $V < 0$ and are thus AdS_4.

- One susy: uplifts to the $D = 11$ solution $AdS_4 \times H^3 \times S^4$ describing the near horizon the the susy wrapped branes.

- One non-susy: uplifts to a distinct $D = 11$ solution $AdS_4 \times H^3 \times S^4$.

Lifshitz invariance in CMT

- Some CM systems (eg some high T_c superconductors) exhibit quantum critical points: phase transitions at $T = 0$.

- At criticality, these systems may display invariance under an anisotropic (Lifshitz) scaling:

$$t \to \lambda^z t \,, \qquad x^i \to \lambda x^i$$

with $z \neq 1$ the dynamical exponent.

- Bottom-up, holographically dual geometry: [Kachru, Liu, Mulligan '09]

$$ds^2 = -r^{2z} dt^2 + r^2 dx^i dx^i + \frac{dr^2}{r^2} \qquad r \to \lambda^{-1} r$$

- Top-down: Some no-go statements about Lifshitz geometries in string/M-theory [Li, Nishioka, Takayangi '09]

Lifshitz geometries in String/M-theory

- Non-explicit constructions [Hartnoll, Polchinski, Silverstein, Tong '09]
- $z = 2$ in type IIB and $D = 11$ sugra. [Balasubramanian, Narayan '10]
- Our $D = 4$ theory contains a Lifshitz solution with $z \sim 39$ dual to a $(1 + 2)$-dim field theory.
- Now also known generalisations of the solution of [Balasubramanian, Narayan '10], also with $z = 2$ [Donos, Gauntlett '10]
- General z in massive type IIA and type IIB [Gregory, Parameswaran, Tasinato, Zavala '10]

Black hole instabilities

- Our $D = 4$ theory can be used to study the $d = 3$ $N = 2$ CFTs on the wrapped M5s at finite T and μ.

- The theory contains RN-AdS black holes at high T. As T decreases two branches of BHs arise:

  - Some neutral modes become unstable in the RN-AdS BH background, providing an M-theory embedding of the set-up of [Goldstein, Kachru, Prakash, Sandip, Trivedi, '09]
  - Some charged modes become unstable in the RN-AdS BH background: new top-down holographic superconductors?

Conclusions and outlook

- We have extended the arena for top-down AdS/CMT applications by studying consistent truncations associated to M5 branes wrapping a Slag_3 cycle in CY_3.
- Instability of on-susy AdS vacua [Bobev, Halmagyi, Pilch, Warner '10]
- Construct fully back-reacted BHs. Superconducting BH thermodynamically preferred over neutral branch?
- What is the ground state of the new branches of BHs, Lifshitz($z \sim 39$)?
- RG flows connecting the different AdS_4 vacua and the Lifshitz solution?
- Aim at a throughout classification of consistently truncated theories.