Structure of the broken phase of the sine-Gordon model

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Introduction[1]

Lagrangian

In Euclidean space time $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + u_\Lambda cos(\beta\phi)$

Main perturbative features at LPA in $d = 2$

- Perturbative UV Coleman fixed point $\beta_c^2 = 8\pi$.
- Phase boundary at the Coleman fixed point.
- Asymptotic freedom of the broken phase $\beta < \beta_c$.
  $\Rightarrow$ the IR is non-perturbative.
- Triviality of the symmetric phase $\beta > \beta_c$.

[1] The following is based on V. Pangon & al. (to appear) Phys Lett B 2010 *hep-th/0907.0496* and V. Pangon *hep-th/1008.0281*

Questions addressed

Physical issues

- The IR strong coupling regime of the SSB phase.
- Universality of the flows
- Generalized universality
- Phase boundary for strongly coupled bare theories.
- (Phase structure in $d \neq 2$)

Methodological issues

- The use of Effective Average Action for periodic models.
- Finding an effective order parameter.
- Representation of the effective potential.
- The convexity problem scalar broken phase.

Derivation

Definitions

- For the bare action $S_\Lambda[\phi]$: $Z[J] = \int \mathcal{D}\phi e^{-S_\Lambda[\phi]-J\cdot\phi}$
- Adding a regulator : $\Delta S_k[\phi] = \frac{1}{2}\int dp\phi(p)\cdot R_k(p)\cdot\phi(-p)$

$$Z_k[J] = \int \mathcal{D}\phi e^{-S_\Lambda[\phi]-\Delta S_k[\phi]-J\cdot\phi}$$

- The Effective Average Action[a] :

$$\begin{aligned} \Gamma_k[\phi] &= min_J\{-LogZ_k[J] - J\cdot\phi\} - \Delta S_k[\phi] \\ k\partial_k\Gamma_k &= \frac{1}{2}Tr\left(\frac{k\partial_k R_k}{\Gamma^{(2)}+R_k}\right) \end{aligned}$$

[a] Wetterich *NPB* (1991), *PLB* (1993),

Properties

Convexity and the broken phase

- The only assumption needed to obtain the flow equation :

$$\Gamma^{(2)} + R_k > 0$$

- At LPA, it reads :

$$\begin{aligned} \overline{k}^2 + V_k'' &> 0 \quad , \quad \overline{k}^2 = min_p\left\{p^2 + R_k(p)\right\} \\ \tilde{k}^2 + \tilde{V}''(\phi_n) &> 0 \quad , \quad \overline{k}^2 = \tilde{k}^2 k^2 \text{in dimensionless} \\ \tilde{k}^2 &= 1 \qquad\quad \text{for } R_k = (k^2 - p^2)\Theta(k^2 - p^2) \end{aligned}$$

- The limit $V_k = -\frac{\overline{k}^2}{2}\phi^2$ is IR attractive in concave regions[a]
  $\Rightarrow$ Possible instability of the flow in the SSB phase.

[a] Tetradis-Wetterich, *NPB* (1992)

Properties

Periodicity

- The bare action $S_\Lambda[\phi]$ is periodic
- Z_k involves a non-periodic contribution ΔS_k at tree-level
- But one-loop computation of the average action gives :

$$\Gamma_k[\phi] \simeq S_\Lambda[\phi] + \frac{1}{2} TrLog\left(S'' + R_k\right)$$

- The initial condition is actually periodic !
- Periodicity is broken only by the "mass" of the fluctuations.
- The genuine flow equation still preserves periodicity :

$$k\partial_k\Gamma_k = \frac{1}{2} Tr\left(\frac{k\partial_k R_k}{\Gamma^{(2)} + R_k}\right)$$

Phase boundary

Flow equation at LPA

$$k\partial_k V_k = \frac{1}{2}\int dp \frac{k\partial_k R_k(p)}{p^2 + R_k(p) + V''}$$

Coleman fixed point

- Linearizing the flow equation ($y = p^2/k^2$, $R_k = p^2 r(p^2/k^2)$)

$$k\partial_k \tilde{V} + 2\tilde{V} = \left(\frac{1}{4\pi}\int_0^\infty dy \frac{r'(y)}{(r(y)+1)^2}\right)\tilde{V}''$$

- All the regulators reproduce the Coleman fixed point :

$$\beta_c^2 = 8\pi\left(-\left[\frac{-1}{r(y)+1}\right]_0^\infty\right)^{-1} = 8\pi$$

Phase boundary

UV flows

$$k\partial_k \tilde{u}(k) \simeq 2\left(\frac{\beta^2}{8\pi}-1\right)\tilde{u}(k) \qquad \text{Potential convex}$$

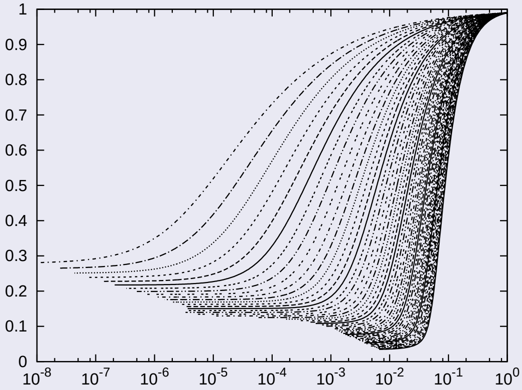
$$k\partial_k u(k) = 2\frac{\beta^2}{8\pi}u(k) \qquad \text{and periodic i. e. constant}$$

Order parameter

- Dimensionless action curvature
  $\tilde{\Gamma}''_k(\phi) = \tilde{k}^2 + \tilde{V}''_k(\phi) \simeq \tilde{k}^2 - \beta^2 \tilde{u}_k cos(\beta\phi)$
- Maximal concavity points $\phi_n = \frac{2n\pi}{\beta}$
- $$\tilde{\Gamma}''_k(\phi_n) \begin{array}{ll} \to \tilde{k}^2 & \text{in the symmetric phase } \beta_r = \beta/\beta_c > 1 \\ \to 0^+ & \text{in the broken phase } \beta_r < 1\,? \end{array}$$
- $$k\partial_k \tilde{\Gamma}''_k(\phi_n) \begin{array}{ll} < 0 & \text{in the symmetric phase } \beta_r > 1 \\ > 0 & \text{in the broken phase } \beta_r < 1 \end{array}$$

Fixed points of the broken phase

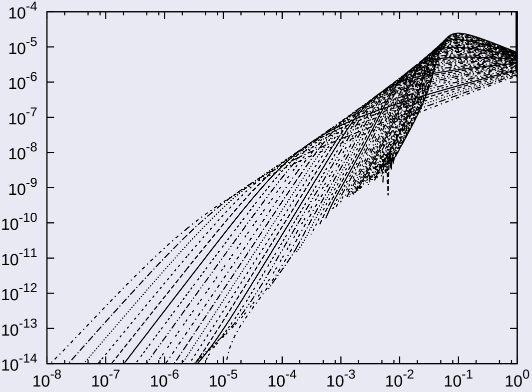
Flow of the curvature in ϕ_n for $\beta_r \in [0.40 : 0.90]$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$



Trajectories always stable ?

Fixed points of the broken phase

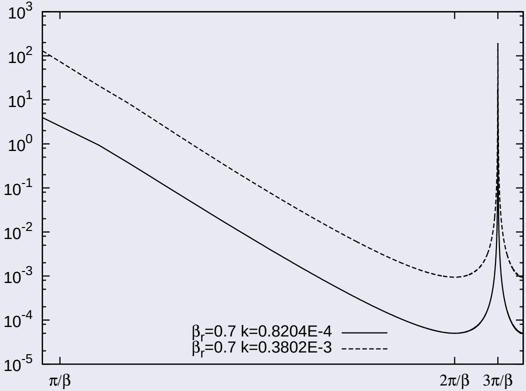
β-function of curvature in ϕ_n for $\beta_r \in [0.40 : 0.90]$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$



Power-law decay towards the fixed points

Fixed points of the broken phase

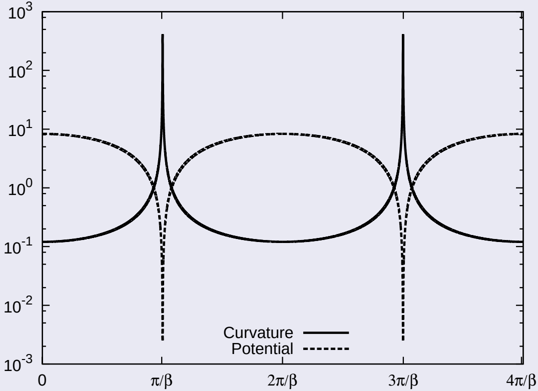
β-function of the curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



Global vanishing of the β-function

Fixed points of the broken phase

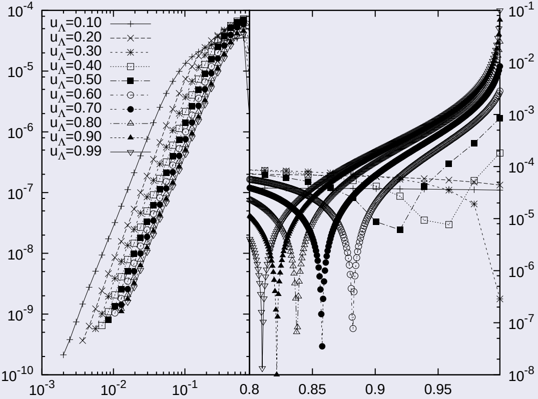
Curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



- Upside-down parabola in concave regions
- Strong restoring force $\Rightarrow$ minimum selection

Universality

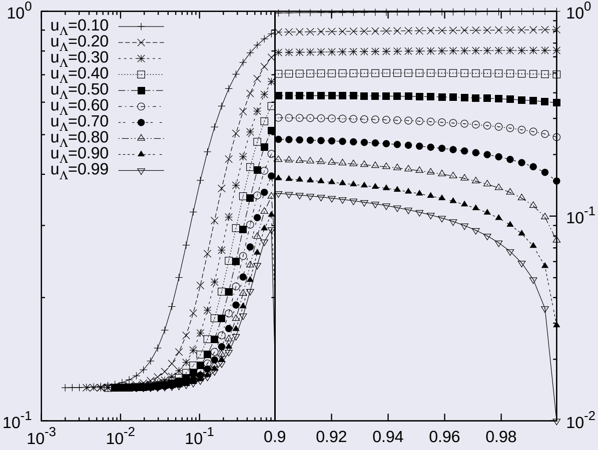
β-function of the curvature in ϕ_n for $\beta_r = 0.70$ for different $\tilde{u}_\Lambda$



- Change of β-function sign in the UV
- Same scaling in the IR

Generalized universality

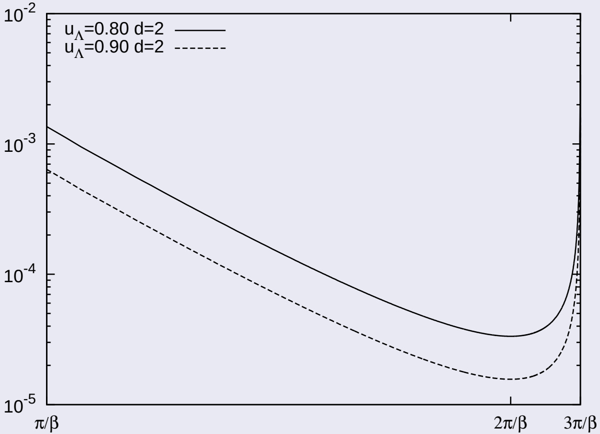
Flow of the curvature in ϕ_n for $\beta_r = 0.70$ for different $\tilde{u}_\Lambda$



IR final value independent of the bare coupling $\tilde{u}$

Generalized universality

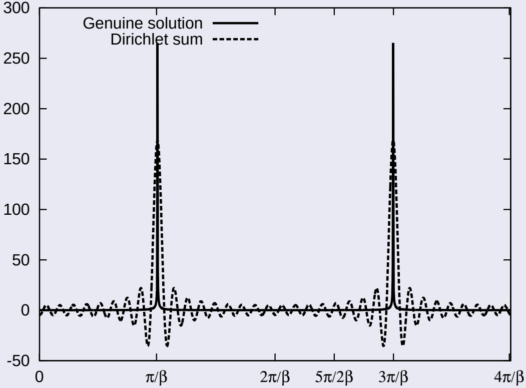
Relative error on the curvature for $\beta_r = 0.70$ for different $\tilde{u}_\Lambda$



The global IR potential is independent of the bare coupling $\tilde{u}$ (ref. $\tilde{u}_\Lambda = 0.01$)

Fourier serie convergence

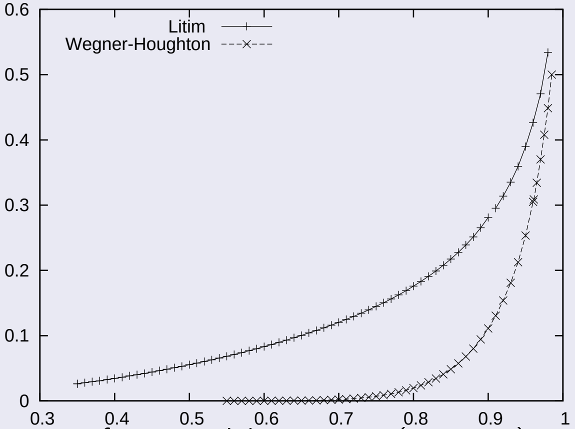
Gibbs phenomenom for the curvature ($\beta_r = 0.70$)



- Non uniform convergence due to almost non-differentiability
- The Parseval sum converges !

Fixed-points line : summary

Fixed-point curvature in ϕ_n for different β_r



Critical exponent for suceptibility $\gamma \simeq 0.4$ (WH $\to 1$)

Smooth cut-off

Power-law results

- Smooth cut-off needed for inclusion of Z.
- The regulator is parametrized :

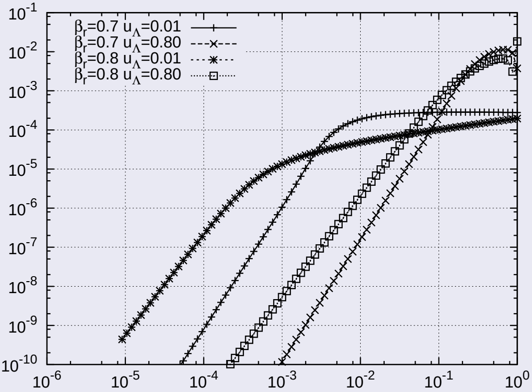
$$r(y) = a_p y^{-b_p},\ (a_p, b_p) \in \mathbb{R}^{+*} \times [1; +\infty]$$

- Special case $b_p = 1$ defines Callan Symanzik RG and matches in $d = 2$ the Wegner-Houghton equation.
- Loop-integral analytical for $b_p = 2$.
- The important quantity for the curvature is :

$$\tilde{k}^2 = (a_p(b_p - 1))^{\frac{1}{b_p}} \left[1 + \frac{1}{b_p - 1}\right] \overset{b_p \to \infty}{\longrightarrow} 1$$

Universality holds

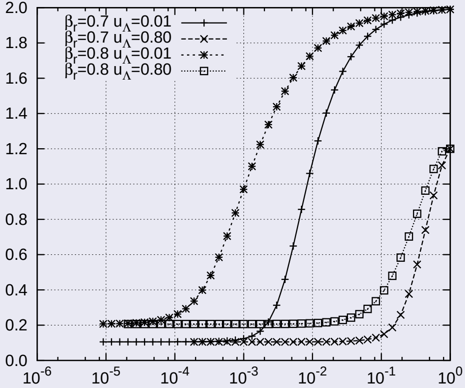
β-function of curvature in ϕ_n



Different power-law decay wrt in Litim case.

Generalized universality holds

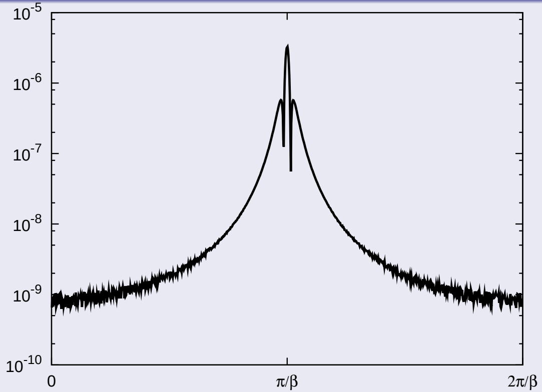
Flow of the curvature in ϕ_n



Different IR ending values wrt in Litim case.

Fixed points of the broken phase

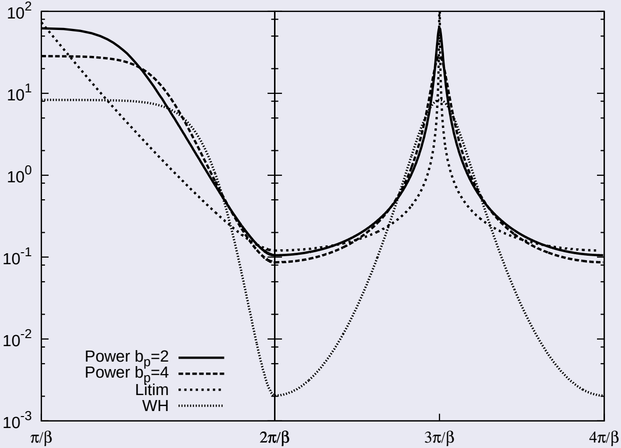
β-function of curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



- Global fixed point.
- Differentiability problem less severe.

Comparison of RG schemes

Curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



Smooth transition from Litim to Wegner-Houghton.

Conclusions

Structure of the broken phase

- It exhibits a line of fixed points for $\beta_r > 0.40$
- No instability in the IR flow
- Generalized universality
- Phase boundary preserved for strongly coupled bare theories
- Bad convergence of the Fourier series

Using Average action

- The Coleman frequency is always reproduced
- All the features of the broken phase are preserved when adding a regulator.
- The qualitative behavior is regulator-independent.

Conclusions

Outlook

- No conceptual problem to study Z
- Possible dependence on the regulator ?
- What happens in other dimensions ?
- Test of propertime flows
- Study of Yang-Mills theories deconfinement transition.
- ...

Thank you for your attention !