The Cosmological Concordance Model of Quantum Field Theory in Curved Spacetime

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C. Dappiaggi, TH, J. Möller, N. Pinamonti, arXiv:1007.5009
TH, arXiv:1008.1776

Quantum Field Theory in Curved Spacetimes

- Quantum field theory on curved spacetimes (QFTCST): matter is a quantum field, the background spacetime is classical
- more *fundamental* than Minkowskian QFT, we can hence expect results that contradict Minkowskian intuition!
- The influence of quantum fields on the background is described by the semiclassical Einstein equation (SEE)

$$G_{\mu\nu} = 8\pi G \langle :T_{\mu\nu}: \rangle_\Omega$$

Quantum Field Theory in Curved Spacetimes (cont)

- The SEE can be ‘derived’ in various ways, a full and satisfactory derivation from Quantum Gravity is not available.
- Pragmatic viewpoint: simplest non-trivial way to investigate matter QFT effects in GR
- GR: everything gravitates. $\rightarrow$ all model-independent QFT effects we find are inevitable!

The role of QFTCST in Cosmology to date

- *Inflation* is thought to be driven by classical fields, their quantized fluctuations play no role in the cosmological evolution.
- Thousands of proposals for *dark energy* exist, but only a very few within QFTCST.
- First inflationary model of Starobinsky (1980) has considered only quantum fields!
- Subsequent works *[Anderson 1983-1986, Sahni et al. 1999, Parker et al. 1999-2006, Shapiro et al. 2000-2009, Koksma 2009, ...]* always ignore some contribution to $\langle :T_{\mu\nu}: \rangle_\Omega$.

Three questions

- This talk: analysis of cosmological implications of QFTCST, taking *all* contributions to $\langle :T_{\mu\nu}: \rangle_\Omega$ into account.

1. Is it possible to model the late cosmological evolution completely within QFTCST?

   Yes.

2. Are there non-trivial QFTCST contributions not present in ΛCDM?

   Yes!

3. Are these contributions significantly visible in experimental data?

   Not yet.

Outline of the remaining talk

1. Computation of $\langle :T_{\mu\nu}: \rangle_\Omega$ in arbitrary spacetimes
2. Analysis of SEE solutions in flat FRW spacetimes
3. Comparison with experimental data

Computation of $\langle :T_{\mu\nu}: \rangle_\Omega$ for free fields in general curved spacetimes

What is a sensible definition of $\langle :T_{\mu\nu}: \rangle$?

- The SEE can only make sense if $\langle :T_{\mu\nu}: \rangle_\Omega$ has finite fluctuations, because $G_{\mu\nu}$ has none!
- $\to \Omega$ has to be a *Hadamard state*, i.e. it has to share the UV properties of the Minkowski vacuum.
- $:T_{\mu\nu}(x):$ should be *local* and *covariant* *[Brunetti, Fredenhagen & Verch 2003, Hollands & Wald 2001]*:
- it should be constructed only by means of the knowledge of the spacetime in a small neighbourhood of x ...
- ... because we should not build our theory on something we don't know, but would like to determine – the full quantum state of all fields in our universe $\cong$ the full spacetime!

The finite regularisation freedom of $\langle :T_{\mu\nu}: \rangle$

- There is no unique local & covariant prescription to define $:T_{\mu\nu}:$ in such a way that $\nabla^\mu :T_{\mu\nu}: = 0$! *[Wald 1978, Hollands & Wald 2005]*

- $\langle :T_{\mu\nu}: \rangle_\Omega$ is determined up to finite, local, conserved curvature tensors.

$$\langle :T_{\mu\nu}(x):' \rangle_\Omega = \langle :T_{\mu\nu}(x): \rangle_\Omega +$$

$$+ \alpha_1 m^4 g_{\mu\nu}(x) + \alpha_2 m^2 G_{\mu\nu}(x) + \alpha_3 I_{\mu\nu}(x) + \alpha_4 J_{\mu\nu}(x)$$

$$I_{\mu\nu} \doteq \frac{1}{\sqrt{|\det g|}} \frac{\delta}{\delta g_{\mu\nu}} \int_M dx \sqrt{|\det g|}\; R^2$$

$$J_{\mu\nu} \doteq \frac{1}{\sqrt{|\det g|}} \frac{\delta}{\delta g_{\mu\nu}} \int_M dx \sqrt{|\det g|}\; R_{\alpha\beta} R^{\alpha\beta}$$

The trace anomaly

- However, in every local regularisation prescription the *trace anomaly* arises *[Wald 1978, Moretti 2003, Hollands & Wald 2005, ...]*:
- the classical stress-energy tensor $T_{\mu\nu}$ fulfils $g^{\mu\nu} T_{\mu\nu} = 0$ if $m = 0$,
- but $g^{\mu\nu} \langle :T_{\mu\nu}: \rangle_\Omega \neq 0$ for $m = 0$!

Analysis of solutions to $G_{\mu\nu} = 8\pi G \langle :T_{\mu\nu}: \rangle_\Omega$ in flat FRW spacetimes

$\langle :T_{\mu\nu}: \rangle_\Omega$ in flat FRW spacetimes

- Flat FRW spacetimes

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2) \qquad H \doteq \frac{\dot{a}}{a}$$

- Matter content:

1. N_0 free conformally coupled real scalar fields with mass m
2. $N_{1/2}$ free Dirac fields with mass m
3. N_1 free massless vector fields

$\langle :T_{\mu\nu}: \rangle_\Omega$ in flat FRW spacetimes (cont)

$$T_Q \doteq g^{\mu\nu}\langle T_{\mu\nu}\rangle_\Omega = -12f\left(\dot{H}H^2 + H^4\right) + \alpha m^4 + \gamma m^2(\dot{H} + 2H^2)$$
$$+\delta(\dddot{H} + 6\ddot{H}H + 4\dot{H}^2 + 12\dot{H}H^2) - N_0 m^2\langle :\phi^2: \rangle_\Omega - N_{1/2} m\langle :\bar{\psi}\psi: \rangle_\Omega$$

$$f \doteq \frac{1}{2880\pi^2}\left(N_0 + 11N_{1/2} + 62N_1\right)$$

[Christensen 1978, Wald 1978, Dappiaggi, TH & Pinamonti 2009]

$$G_{\mu\nu} = 8\pi G\langle :T_{\mu\nu}: \rangle_\Omega \quad \cong \quad \nabla^\mu\langle :T_{\mu\nu}: \rangle_\Omega \quad \text{and} \quad 3H^2 = 8\pi G\rho_Q$$

$$\frac{\dot{\rho}_Q}{H} + 4\rho_Q = -T_Q$$

The state dependence

- We fix the state Ω to be an approximate thermal equilibrium state with $\beta = T^{-1}$ at $a = a_0$ *[Dappiaggi, TH, Pinamonti 2010, TH 2010]*, e.g.

$$\langle\phi(\tau_x,\vec{x})\phi(\tau_y,\vec{y})\rangle_\Omega = \int\limits_{\mathbb{R}^3} d\vec{k} \left(\frac{\overline{T_k(\tau_x)}\,T_k(\tau_y)}{1-e^{-\beta\sqrt{k^2+a_0^2m^2}}} + \frac{\overline{T_k(\tau_y)}\,T_k(\tau_x)}{e^{\beta\sqrt{k^2+a_0^2m^2}}-1}\right) e^{i\vec{k}(\vec{x}-\vec{y})}$$

$$\left(\partial_\tau^2 + k^2 + a^2m^2\right) aT_k(\tau) = 0 \qquad aT_k(\tau) \sim \frac{e^{-ik\tau}}{\sqrt{2\pi}^3\sqrt{2k}} \quad \text{for} \quad a \to 0$$

- This state is Hadamard and models the quantum state of (dark) matter after the *freeze-out* of the (dark) matter interactions at $a = a_0$.

A few necessary approximations

- Exact analytical computations of $\langle :\phi^2: \rangle_\Omega$, $\langle :\bar{\psi}\psi: \rangle_\Omega$, and the solutions of the SEE are impossible.

- We employ the following approximations to investigate SEE solutions describing the late cosmological evolution:

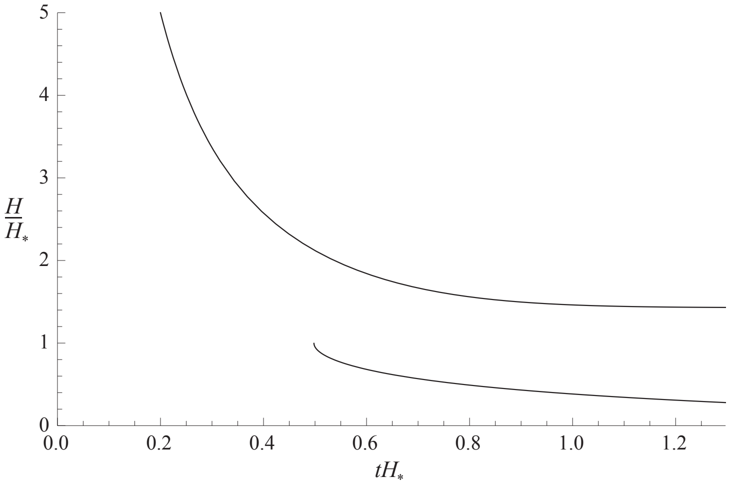
  1. $H \ll m$ $[H_0 = O(10^{-33} eV)]$ $\rightarrow$ discard $O(H^5)$ terms in ϱ_Q
  2. $\dot{H} \ll H^2$ [observations] $\rightarrow$ discard $O(\dot{H}, \ddot{H}, \ldots)$ terms in ϱ_Q
  3. $T \ll am$ [dark matter is cold]

The solutions

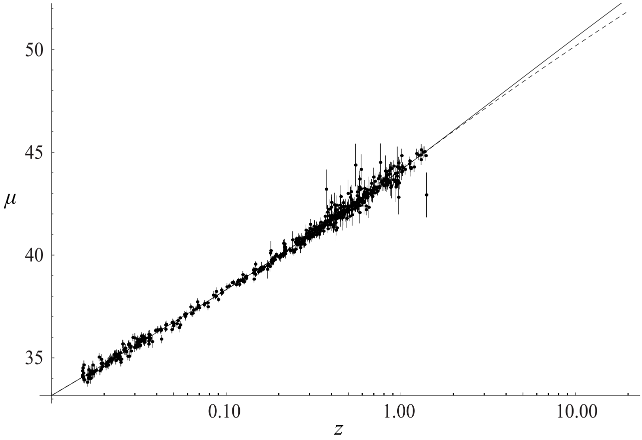
$$\rho_Q = c_1 H^4 + c_2 \frac{mT^3}{a^3} + c_3 \frac{T^5}{ma^5} + \frac{c_4}{a^4} + c_5 m^4 + c_6 H^2 + O\left(\frac{T^7}{m^3 a^7}\right)$$

$$H_\pm^2(a) = H_*^2 \pm \sqrt{H_*^4 - C_2 \frac{mT^3}{a^3} - C_3 \frac{T^5}{ma^5} - \frac{C_4}{a^4} - C_5 m^4}$$

Plot of generic solutions with $\dot{H} < 0$

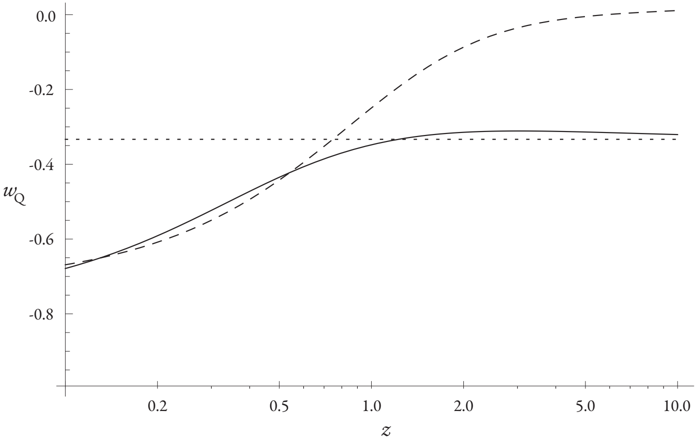


Comparison with experimental data



Union2 supernova compilation *[Amanullah et al. 2010]*

Both solution branches can fit supernova data equally well.



But there is no significant matter-dominated phase on the upper branch.
$(w_Q = p_Q/\rho_Q)$

Non-trivial quantum corrections to ΛCDM

- For the lower branch, the supernova fits generally yield values for which $H_-^2(a)$ can be expanded in H_*^{-1}.

$$H_-^2(a) = H_*^2 \pm \sqrt{H_*^4 - C_2 \frac{mT^3}{a^3} - C_3 \frac{T^5}{ma^5} - \frac{C_4}{a^4} - C_5 m^4}$$

$$= K_0 + \frac{K_3}{a^3} + \frac{K_4}{a^4} + \frac{K_5}{a^5} + O\left(\frac{1}{a^6}\right)$$

- For K_0 and K_3, the best supernova fits yield standard ΛCDM values, whereas the current data is insensitive to quantum corrections to ΛCDM as large as $K_5 = O(10^{-3} K_0)$

Conclusions

Bringing in the harvest

We have seen ...

- ... that it is possible to reproduce the phenomenology of ΛCDM from first QFTCST principles.
- Non-trivial quantum corrections to ΛCDM arise, which entail that *dark matter* has a different scaling behaviour w.r.t. *a* and that *dark energy* is dynamical!
- The currently available data is insensitive to these quantum corrections, but we can expect an exponential increase in (supernova) data in the next decade.

Thanks a lot for your attention!