Asymptotically Safe Gravity

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DFG

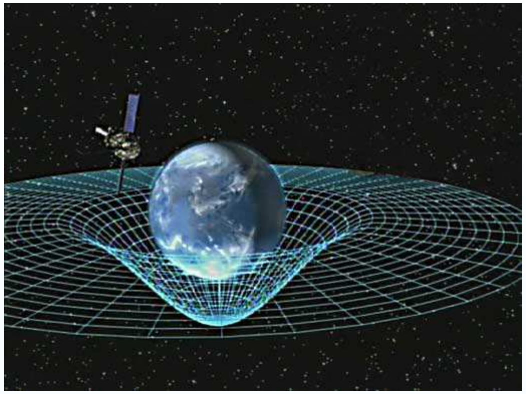
DESY Workshop

Quantum Field Theory: Developments and Perspectives

September 23th, 2010

Classical General Relativity

Based on Einsteins equations



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- Newton's constant: $G_N = 6.67 \times 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$
- cosmological constant: $\Lambda \approx 10^{-35}\ \mathrm{s}^{-2}$

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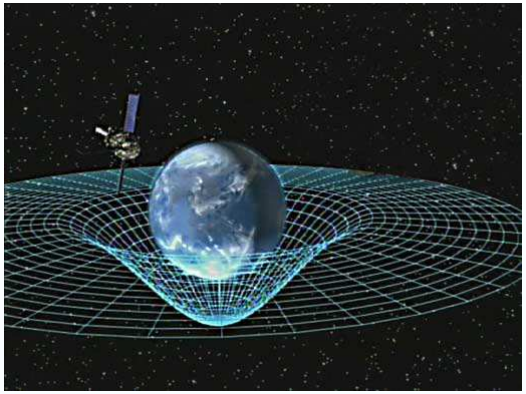

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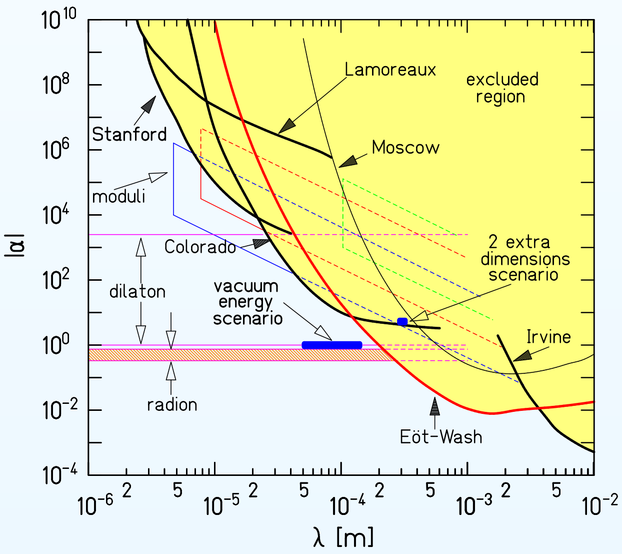

[Adelberger, et. al., 2004]

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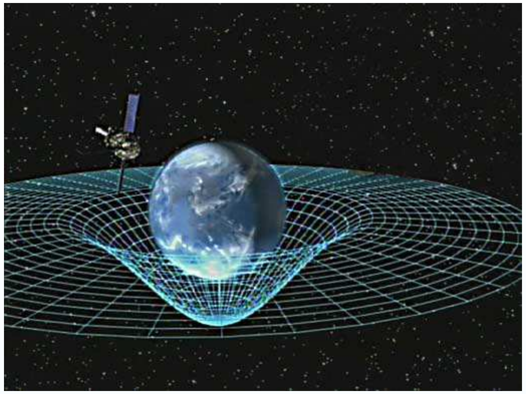

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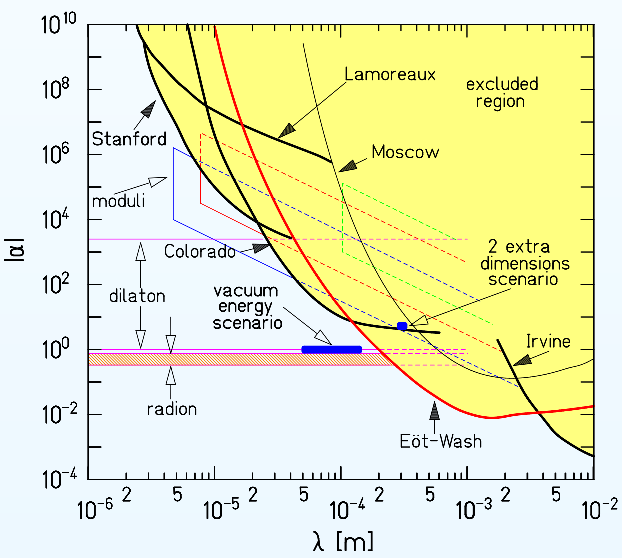

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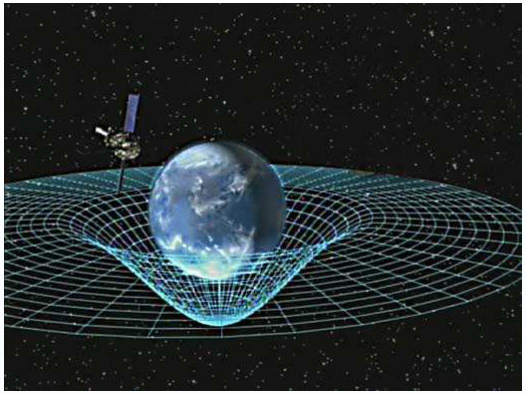

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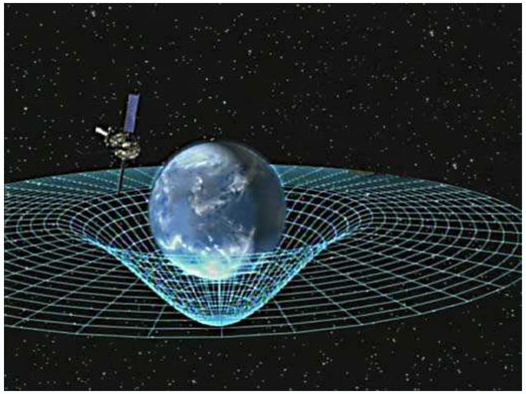

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Theoretical guidance: Quantum Theory for Gravity

Gravity: Perturbative quantization

Length scale for Quantum Gravity Effects:

Planck scale: $$\ell_{\mathrm{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\mathrm{cm}\,;\; m_{\mathrm{Pl}} = 10^{19}\mathrm{GeV}$$

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perturbative quantization of General Relativity

$$S^{\rm EH} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,\{-R+2\Lambda\}$$

- G_N has negative mass-dimension $\Leftrightarrow$ infinite number of counterterms

gravity + scalar: $\Delta S^{1-{\rm loop}} \propto \int d^4x\sqrt{g}\,\left\{C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu}\right\}$

pure gravity: $\Delta S^{2-{\rm loop}} \propto \int d^4x\sqrt{g}\,\left\{C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\alpha\beta}C_{\alpha\beta}{}^{\mu\nu}\right\}$

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Possible conclusions:

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Renormalizing

the Non-Renormalizable

Wilson's modern picture of renormalization

Concept: describe physical system by family of effective actions valid at given scale k

implementation:

- action with scale-dependent couplings $(G_N, \Lambda, \dots)$: $g_i(k)$
- scale-dependence governed by β-functions: $k\partial_k g_i = \beta_{g_i}(\{g_i\})$
- central: fixed points of RG-flow: $\beta_{g_i}(\{g_i\}) \stackrel{!}{=} 0$

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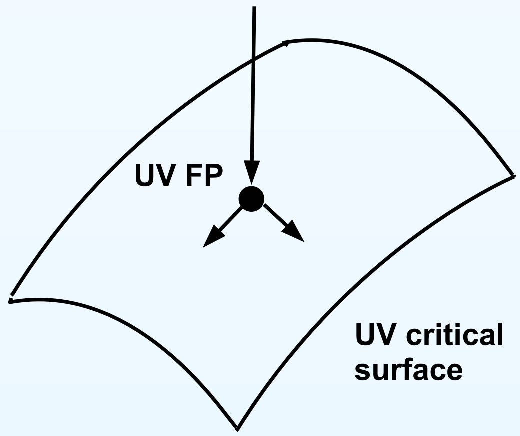
UV fixed points

- well-defined continuum limit
  - trajectory captured by fixed point has no unphysical UV divergences
- classes of trajectories
  - stable, unstable
  - unstable directions = UV critical surface
- predictivity
  - finite number of unstable directions



Renormalization: asymptotic freedom and asymptotic safety

Wilsonian formulation:

- UV fixed points allow two classes of renormalizable Quantum Field Theories
- Gaussian Fixed Point
  - perturbatively renormalizable field theories
  - fundamental theory: free
  - asymptotic freedom (example: QCD)

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Wilsonian picture: generalization of perturbative renormalization

asymptotic safety as predictive as asymptotic freedom

Examples for Asymptotically Safe Theories

Examples: theories with non-Gaussian ultra-violet fixed point

- $O(N)$-sigma model

  (Berezin, Zinn-Justin ’76)

  - $N = 2, d = 4 - \epsilon$ has non-trivial UV fixed point
  - critical exponents $\Leftrightarrow$ heat exponent of superfluid Helium

- Gross-Neveu model ($d = 2 + \epsilon$)

  (Gawedzki, Kupiainen ’85)

- Gravity in $2 + \epsilon$ dimensions

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Asymptotic safety scenario for gravity

Gravitational coupling in d dimensions: $g_k = k^{d-2} G_N(k)$

- RG scaling of gravitational coupling

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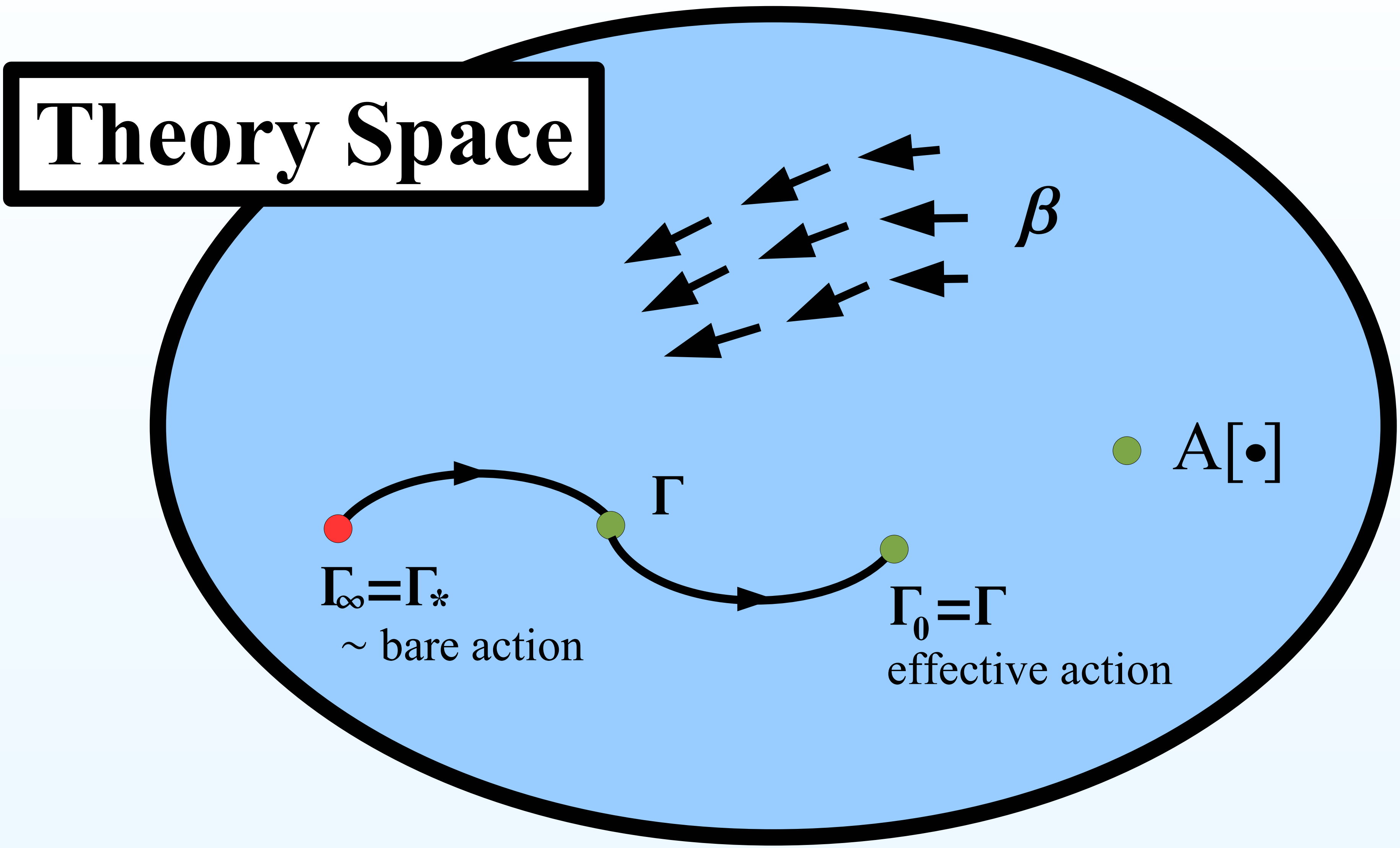
Goal: Show that non-Gaussian Fixed Point exists

Functional Renormalization Group Equations

Theory space underlying the Functional Renormalization Group



Functional Renormalization Group Equation for gravity

Flow equation for effective average action Γ_k

(C. Wetterich, Phys. Lett. $\mathbf{B301}$ (1993) 90)

generalized to gravity

(M. Reuter, Phys. Rev. D $\mathbf{57}$ (1998) 971, hep-th/9605030)

$$k\partial_k \Gamma_k[g, C, \bar{C}; \bar{g}, c, \bar{c}] = \frac{1}{2}\mathrm{STr}\left[\left[\Gamma_k^{(2)} + \mathcal{R}_k\right]^{-1} k\partial_k \mathcal{R}_k\right]$$

- k: coarse-graining scale
- $\mathcal{R}_k$: mass-term for fluctuations with $p^2 \ll k^2$
- background-covariance: ensured by background field formalism

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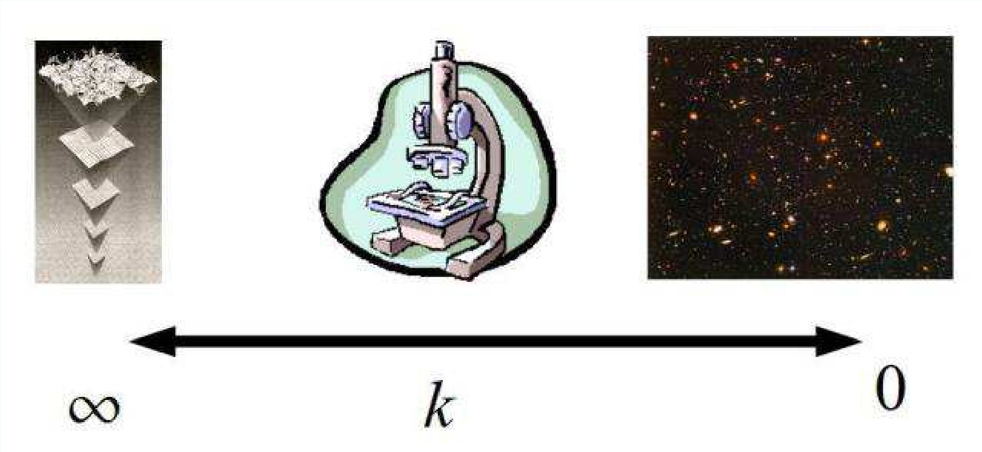

Non-perturbative approximation: Derivative expansion of Γ_k

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- Expand Γ_k in derivatives and truncate series:

$$\Gamma_k[\Phi] = \sum_{i=1}^{N} \bar{u}_i(k)\, \mathcal{O}_i[\Phi]$$

  - $\Longrightarrow$ substitute into FRGE
  - $\Longrightarrow$ projection of flow gives β-functions for running couplings

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- Testing the reliability:
  - within a given truncation:
    - cutoff-scheme dependence of physical quantities (= vary $R_k^{(0)}$)
  - stability of results under extensions of the truncation

Einstein-Hilbert Results

Charting the theory space spanned by $\Gamma_k^{\rm grav}[g]$

Einstein-Hilbert truncation

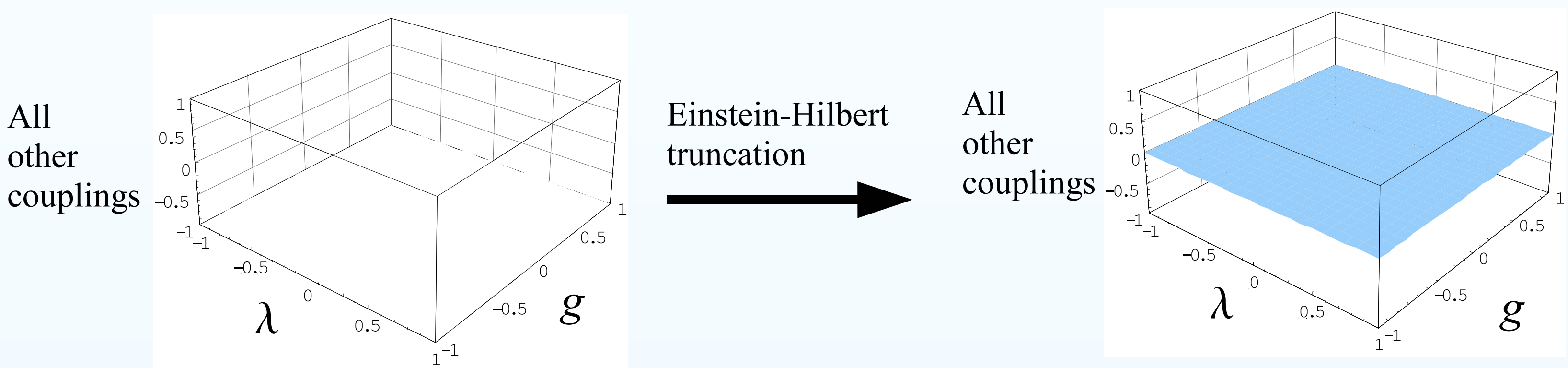
$\vdots$			
R^8	$\ldots$		
R^7	$\ldots$		
R^6	$\ldots$		
R^5	$\ldots$		
R^4	$\ldots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$	$R \Box R$	+ 7 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
$\mathbb{1}$			

The Einstein-Hilbert truncation: setup

Einstein-Hilbert truncation: two running couplings: $G(k), \Lambda(k)$

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x\sqrt{g}\,[-R + 2\Lambda(k)] + \dots$$

- project flow onto truncation subspace



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explicit β-functions for dimensionless couplings $g_k := k^2 G(k)\,,\ \lambda_k := \Lambda(k) k^{-2}$

- Particular choice of R_k (optimized cutoff)

$$k\partial_k g_k = (\eta_N + 2) g_k\,,$$

$$k\partial_k \lambda_k = -\,(2 - \eta_N)\,\lambda_k - \frac{g_k}{2\pi}\left[5\,\frac{1}{1-2\,\lambda_k} - 4 - \frac{5}{6}\,\frac{1}{1-2\lambda_k}\eta_N\right]$$

- anomalous dimension of Newton's constant:

$$\eta_N = \frac{gB_1}{1 - gB_2}$$

$$B_1 = \frac{1}{3\pi}\left[5\,\frac{1}{1-2\lambda} - 9\,\frac{1}{(1-2\lambda)^2} - 7\right]\,,\ B_2 = -\frac{1}{12\pi}\left[5\,\frac{1}{1-2\lambda} + 6\,\frac{1}{(1-2\lambda)^2}\right]$$

Einstein-Hilbert truncation: Fixed Point structure

- β-functions for dimensionless couplings: $g_k := k^2 G(k)\,,\ \lambda_k := \Lambda(k) k^{-2}$

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  - UV attractive in g_k, λ_k

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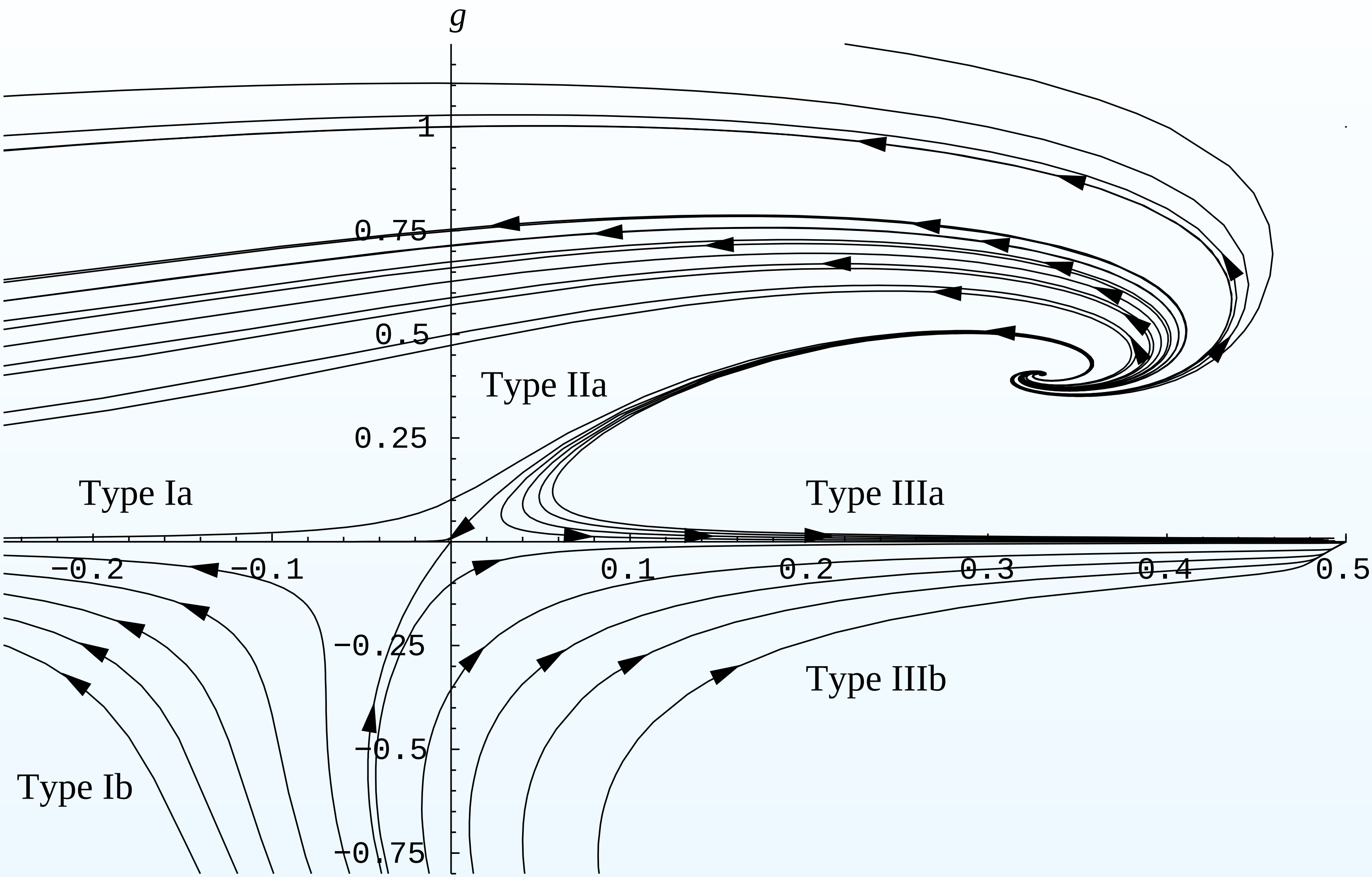
Asymptotic safety: non-Gaussian Fixed Point is UV completion for gravity

Einstein-Hilbert truncation: phase diagram



Einstein-Hilbert truncation: Stability properties

Ref.	g^*	λ^*	$g^*\lambda^*$	$\theta' \pm i\theta''$	gauge	$\mathcal{R}_k$
BMS	0.902	0.109	0.099	$2.52 \pm 1.78i$	geometric	II, opt
RS	0.403	0.330	0.133	$1.94 \pm 3.15i$	harmonic	I, sharp
LR	0.272	0.348	0.095	$1.55 \pm 3.84i$	harmonic	I, exp
	0.344	0.339	0.117	$1.86 \pm 4.08i$	Landau	I, exp
L	1.17	0.25	0.295	$1.67 \pm 4.31i$	Landau	I, opt
CPR	0.707	0.193	0.137	$1.48 \pm 3.04i$	harmonic	I, opt
	0.556	0.092	0.051	$2.43 \pm 1.27i$	harmonic	II, opt
	0.332	0.274	0.091	$1.75 \pm 2.07i$	harmonic	III, opt

BMS: Benedetti, Machado, Saueressig, 2009.

RS: Reuter, Saueressig, 2002.

LR: Lauscher, Reuter, 2002.

L: Litim, 2004.

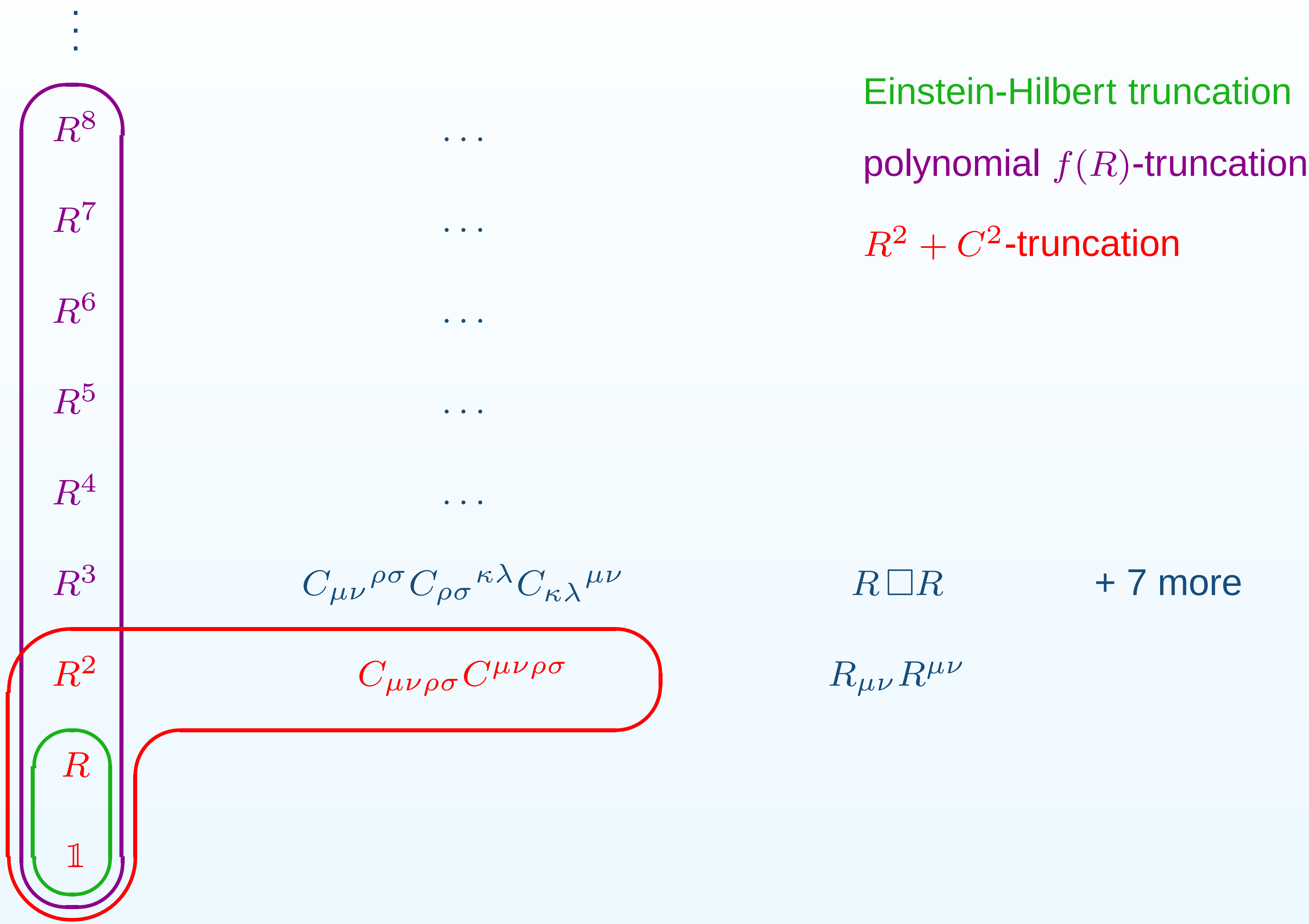
CPR: Codello, Percacci, Rahmede, 2009.

Einstein-Hilbert truncation: Open Questions

- Existence of NGFP in extended truncations?
- How many couplings are relevant?
- What about . . . perturbative counterterms?

Higher-Derivative Gravity . . .

Charting the theory space of gravity



The $R^2 + C^2$-Truncation

Truncation ansatz

$$\Gamma_k^{\text{grav}}[g] = \int d^4x \sqrt{g} \left[\bar{u}_0 + \bar{u}_1 R + \bar{u}_2 R^2 + \bar{u}_3 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right]$$

- C^2-coupling: characteristic behavior of Higher-Derivative gravity

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computing β-functions: Novel projection technique for traces

- $f(R)$-computation: maximally symmetric background metric (S^d)
  - insufficient to disentangle running of $\bar{u}_2$ and $\bar{u}_3$!
- evaluate traces on generic Einstein background
  - background sufficiently general to distinguish R^2 and C^2 interaction!
  - all differential operators are Lichnerowicz:

$$\Delta_{2,L}\phi_{\mu\nu} = -D^2\phi_{\mu\nu} - 2R_\mu{}^\alpha{}_\nu{}^\beta\phi_{\alpha\beta}$$

$$\Delta_{1,L}\phi_\mu = -D^2\phi_\mu - R_{\mu\nu}\phi^\nu$$

$$\Delta_{0,L}\phi = -D^2\phi$$

  - Trace-evaluation: standard heat-kernel techniques

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Structure of the flow equation:

$$k\partial_k \Gamma_k[\Phi] = \mathcal{S}_{2\text{T}} + \mathcal{S}_{hh} + \mathcal{S}_{1\text{T}} + \mathcal{S}_0$$

- gravity contribution:

$$\mathcal{S}_{2\text{T}} = \tfrac{1}{2} \text{Tr}_{2\text{T}} \left[\frac{\partial_t \left\{ 2\bar{u}_3 (P_{2,k}^2 - \Delta_{2,L}^2) - (\bar{u}_1 + \bar{u}_\flat R) R_{2,k} \right\}}{2\bar{u}_3 P_{2,k}^2 - (\bar{u}_1 + \bar{u}_\flat R) P_{2,k} - \frac{1}{2} \bar{u}_1 R - \bar{u}_0} \right] ,$$

$$\mathcal{S}_{hh} = \tfrac{1}{2} \text{Tr}_0 \left[\frac{\partial_t \left\{ 6\bar{u}_2 (P_{0,k}^2 - \Delta_{0,L}^2) + (\bar{u}_1 - 2\bar{u}_2 R) R_{0,k} \right\}}{6\bar{u}_2 P_{0,k}^2 + (\bar{u}_1 - 2\bar{u}_2 R) P_{0,k} + \frac{2}{3} \bar{u}_0} \right]$$

- universal contribution (auxiliary and ghost fields):

$$\mathcal{S}_{1\text{T}} = -\tfrac{1}{2} \text{Tr}_{1\text{T}} \left[\frac{\partial_t R_{1,k}}{P_{1,k}} \right] , \quad \mathcal{S}_0 = -\tfrac{3}{2} \text{Tr}_0 \left[\frac{\partial_t R_{0,k}}{3P_{0,k} - R} \right]$$

NGFP in the $R^2 + C^2$-Truncation

Truncation	g^*	λ^*	g_2	g_3	g_4	$\lambda^* g^*$	cutoff
$R^2 + C^2$	1.960	0.218	0.008	-0.0050	–	0.427	non-pert.
LR II	0.292	0.330	0.005	–	–	0.096	non-pert.
CP	1.389	0.221	$*$	$*$	$*$	0.307	one-loop

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- Stability properties: (lift degeneracy of marginal couplings)

$$\theta_0 = 2.51\,, \quad \theta_1 = 1.69\,, \quad \theta_2 = 8.40\,, \quad \theta_3 = -2.11$$

  - 3 UV-attractive + 1 UV-repulsive eigendirection

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- RG trajectory captured by NGFP in UV $\Longleftrightarrow$ condition between couplings
  - linear regime around NGFP:

$$g_3 = -0.116 + 0.745 g_0 - 2.441 g_1 + 11.06 g_2$$

flow towards IR $\Leftrightarrow$ relation between couplings in effective field theory

Higher-Derivative Gravity

and perturbative counterterms

Renormalizing the non-renormalizable: $R^2 + C^2$+scalar field

Prototype of gravitational theory: perturbatively non-renormalizable at one-loop

$$\Gamma_k[\Phi] = \Gamma_k^{\mathrm{grav}}[g] + \Gamma^{\mathrm{sf}}[g,\phi] + S^{\mathrm{gf}}[g;\bar{g}] + S^{\mathrm{gh}}[g,\bar{g},C,b]$$

- gravitational and matter sectors:

$$\Gamma_k^{\mathrm{grav}}[g] = \int d^4x\sqrt{g}\left[\frac{1}{16\pi G_k}(-R+2\Lambda_k) - \frac{\omega_k}{3\sigma_k}R^2 + \frac{1}{2\sigma_k}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right]$$

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- Perturbative viewpoint:
  - "fundamental theory"
  - "perturbative counterterms" = non-renormalizable one-loop divergencies
- Functional Renormalization group result:

  (Percacci, Perini, Phys.Rev.D67 (2003) 081503)

  - Truncation has NGFP = as pure gravity

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Influence of perturbative counterterms on asymptotic safety?

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- matter contribution

$$\mathcal{S}_{\rm sf} = \frac{1}{2}{\rm Tr}_0\left[\frac{\partial_t R_{0,k}}{P_{0,k}}\right]$$

- computation of β-functions: completely analogous to pure gravity!

NGFP for gravity coupled to scalar matter

pure gravity:

Truncation	g^*	λ^*	g_2	g_3	$g^*\lambda^*$
Einstein-Hilbert	0.902	0.109	–	–	0.099
$R^2 + C^2$	1.960	0.218	0.008	-0.0050	0.427

gravity minimally coupled to free scalar (includes perturbative divergences!):

Truncation	g^*	λ^*	g_2	g_3	$g^*\lambda^*$
Einstein-Hilbert	0.860	0.131	–	–	0.112
$R^2 + C^2$+scalar	2.280	0.251	0.010	-0.0043	0.571

gravity-matter NGFP persists upon inclusion of perturbative counterterms

Stability properties of the NGFP

pure gravity:

Truncation	θ_0	θ_1	θ_2	θ_3
Einstein-Hilbert	$2.52 + 1.78i$	$2.52 - 1.78i$	–	–
$R^2 + C^2$	2.51	1.69	8.40	-2.11

gravity minimally coupled to free scalar (includes perturbative divergences!):

Truncation	θ_0	θ_1	θ_2	θ_3
Einstein-Hilbert	$2.58 + 1.95i$	$2.58 - 1.95i$	–	–
$R^2 + C^2$+scalar	2.68	1.39	7.89	-1.49

gravity-scalar theory inherits asymptotic safety from pure gravity

Summary

pure gravity:

- all truncation data points at existence of NGFP:
    - higher derivative C^2-truncation complements $f(R)$-results!
- $\Gamma_k^{\rm grav}[g]$: 3-dimensional UV critical surface of NGFP:
    - relation between gravitational couplings at 4-derivative level

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- gravity-matter theories inheriting asymptotic safety from gravity!
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Strong results in favor of asymptotic safety

Still: Many things to explore