Effective theory methods for heavy coloured (s)particles at hadron colliders

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(Based on M.Beneke, P.Falgari, CS, arXiv:0907.1443 [hep-ph], arXiv:1007.5414 [hep-ph]

M.Beneke, M.Czakon, P.Falgari, A.Mitov, CS arXiv:0911.5166 [hep-ph]

M.Beneke, P.Falgari, S. Klein, CS, in progress)

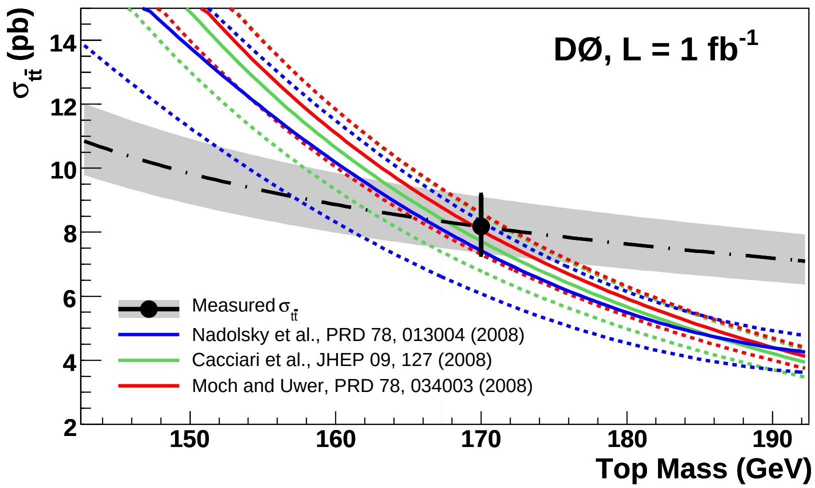
Pair production of heavy coloured particles at Tevatron/LHC

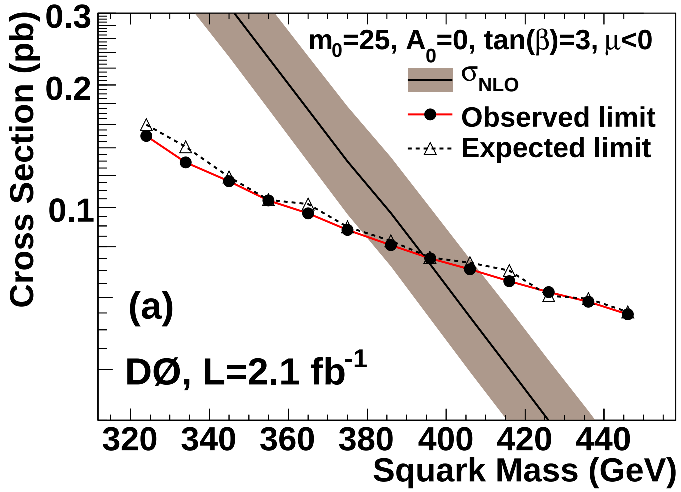
$$N(K_1)N'(K_2) \to H(p_1)H'(p_2) + X$$

- N, N': pp, $p\bar{p}$; HH': top-quark, squark, gluino... pairs

Precise knowledge of total cross sections:

- top-quarks: sensitivity on mass, constraining gluon PDFs
- new particles: Exclusion bounds, model discrimination,...





NLO corrections enhanced for $\beta = \sqrt{1 - \frac{(M_H + M_{H'})^2}{\hat{s}}} \rightarrow 0$

$$\hat{\sigma}^{(1)}_{pp' \rightarrow HH'} = \hat{\sigma}^{(0)}_{pp' \rightarrow HH'}\, \alpha_s \left[\underbrace{a \log^2(8\beta^2) + b \log(8\beta^2)}_{\text{“threshold logarithms”}} + \underbrace{c \frac{1}{\beta}}_{\text{“Coulomb singularity”}} + \ldots \right]$$

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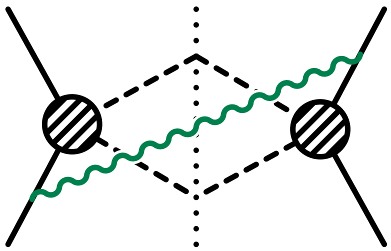
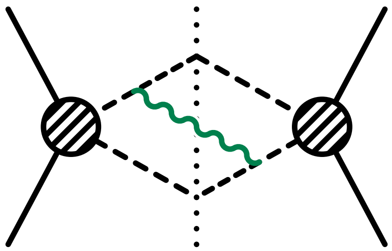
Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, . . .)

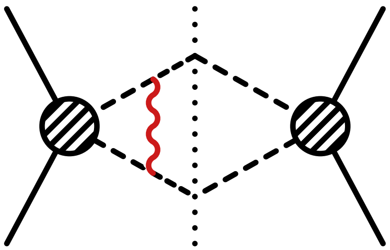


$\Rightarrow \alpha_s \; \log^2(8\beta^2)$



$\Rightarrow \alpha_s \; \log(8\beta^2)$

Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,. . .)



$\Rightarrow \alpha_s \; \frac{1}{\beta}$

Combination of Coulomb- and soft effects? Not trivial:

- Heavy particles nonrelativistic near threshold:

$$E \sim m\beta^2 \,, \quad |\vec{p}| \sim m\beta$$

- soft gluon momenta of same order: $q_s \sim m\beta^2 \sim E$
  $\Rightarrow$ heavy particles "feel" soft radiation

Factorization into hard, soft and Coulomb functions:

$$\hat{\sigma}_{pp' \to HH'}|_{\hat{s} \to 4M^2} = H_{ij} \otimes W_{ij} \otimes J$$

using EFTs for collinear p, p', non-relativistic HH', soft gluons

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Application to top quark, squark and gluino production

- Diagonal colour bases for $3 \otimes 3$, $3 \otimes \bar{3}$, $3 \otimes 8$, $8 \otimes 8$
- NLL results for squark-antisquark production
- $\mathcal{O}(\alpha_s^4)$ threshold expansion +NNLL resummation for $t\bar{t}$

LO NRQCD Lagrangian for particles H, H' in representations R, R':

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi'$$
$$+ \int d^3\vec{r} \left[\psi^\dagger \mathbf{T}^{(R)a} \psi \right](\vec{r}) \left(\frac{\alpha_s}{r} \right) \left[\psi'^\dagger \mathbf{T}^{(R')a} \psi' \right](0) \,,$$

with $D_s^0 = \partial^0 - i g_s A_s^0(x_0, \vec{0})$.

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with $D_s^0 = \partial^0 - ig_s A_s^0(x_0, \vec{0})$.

Decoupling for heavy particle fields:

$$\psi(x) = S_v^{(R)}(x_0)\psi^{(0)\dagger}(x), \quad S_v^{(R)}(x) = \overline{\mathrm{P}} \exp \left[-ig_s \int_0^\infty ds\, v \cdot A^a(x + vs) \mathbf{T}^{(R)a} \right]$$
$$\Rightarrow D_s^0 \psi = S_v \partial^0 \psi^0$$

same $v = (1, \vec{0})$ for both heavy particles at threshold

Works at leading order in PNRQCD

(sufficient at NNLL: Beneke, Czakon, Falgari, Mitov, CS 09; Beneke, Falgari, CS 10)

Analogous redefinition in SCET (Bauer, Pirjol, Stewart 01)

Apply soft-gluon decoupling to amplitude:

$$\mathcal{A}_{pp'\to HH'X} \Rightarrow \sum_i C^{(i)} \langle HH'|\psi^{(0)\dagger}\psi'^{(0)\,\dagger}|0\rangle\ \langle 0|\phi_c^{(0)}|p\rangle\langle 0|\phi_{\bar{c}}^{(0)}|p'\rangle\ \langle X|S_n S_{\bar{n}} \boldsymbol{c}^{(i)} S_v^\dagger S_v^\dagger|0\rangle$$

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Inserting into formula for σ, summing over complete set of $|X\rangle\ldots$

$$\hat{\sigma}_{pp'}(\hat{s},\mu) = \sum_{i,i'} H_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(\sqrt{\hat{s}} - 2M - \frac{\omega}{2}) W_{ii'}^{R_\alpha}(\omega,\mu)$$

Irreducible representations $R \otimes R' = \sum_{R_\alpha} R_\alpha$ e.g. $3 \otimes \overline{3} = 1 \oplus 8$.

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Irreducible representations $R\otimes R' = \sum_{R_\alpha} R_\alpha$ e.g. $3\otimes\bar{3} = 1\oplus 8$.

Soft function:

$$W_{ii'}^{R_\alpha}(\omega) = \int \frac{dz_0}{4\pi}\, e^{i\omega z_0/2}\, \langle 0|\overline{\mathrm{T}}[S_v S_v c^{(i')*} S_{\bar{n}}^\dagger S_n^\dagger](0) P^{R_\alpha}\, \mathrm{T}[S_n S_{\bar{n}} c^{(i)} S_v^\dagger S_v^\dagger](x_0)|0\rangle$$

Potential function:

$$J_{R_\alpha}(E) = \int d^4z e^{iEz^0}\, \langle 0|[\psi^{(0)}\psi'^{(0)}](z^0) P^{R_\alpha} [\psi^{(0)\dagger}\psi'^{(0)\dagger}](0)|0\rangle \quad = 2\mathrm{Im} G_C^{R_\alpha}(0,0,E)$$

(Same as for $e^-e^+\to t\bar{t}$: Fadin, Khoze 87; Beneke, Signer, Smirnov; Hoang, Teubner 99,...)

Construction of colour basis tenors $c^{(i)}$

for all squark/gluino production processes $\tilde{q}\bar{\tilde{q}}$, $\tilde{q}\tilde{q}$, $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{g}$

- soft radiation off total colour charge of $(HH')_{R_\alpha}$ (Bonciani et.al. 98)
- one-loop basis (Kidonakis/Sterman 97; Kulesza/Moytka 08, Beenakker et.al. 09)
- all-order diagonalization of soft function (Beneke, Falgari, CS 09)
  - pairs of equivalent initial- and final state representations:

    e.g. $8 \otimes 8 \to 3 \otimes \bar{3}:\qquad P_i \in \{(1,1),\ (8_S,8),\ (8_A,8)\}$
  - Clebsch-Gordan coefficients e.g. $8 \otimes 8 \to 8_A$, $3 \otimes \bar{3} \to 8$:

    $$C^{(8_A)}_{\alpha a_1 a_2} = \frac{i}{\sqrt{3}} f^{a_2 \alpha a_1}, \qquad C^{(8)}_{\alpha a_1 a_2} = \sqrt{2} T^{\alpha}_{a_2 a_1}$$
  - construct basis tensors:

    $$c^{(i)}_{\{a\}} = \frac{1}{\sqrt{\text{dim}(r_\alpha)}} C^{r_\alpha}_{\alpha a_1 a_2} C^{R_\beta *}_{\alpha a_3 a_4} \qquad \text{e.g. } c^{(3)}_{\{a\}} = \frac{i}{\sqrt{12}} f^{a_2 \alpha a_1} T^{\alpha}_{a_3 a_4}$$
- Combine final-state Wilson lines: $C^{R_\beta} S_v^{(R)} S_v^{(R')} = S_v^{R_\beta} C^{R_\beta}$

$\Rightarrow$ soft function for single final-state particle in R_α

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation
- $\frac{dH_i}{d\mu} \Rightarrow$ related to IR singlarities (Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ get RGE for soft function

$$\frac{d}{d\log\mu} W_i^{R_\alpha}(z^0,\mu) = \Big(2\gamma_{\text{cusp}}(C_r + C_{r'})\log\left(\frac{iz_0\bar{\mu}}{2}\right) - 2(\gamma_{H.s}^{R_\alpha} + \overbrace{\gamma_s^r + \gamma_s^{r'}}^{\text{as for Drell-Yan/Higgs}})\Big) W_i^{R_\alpha}(z^0,\mu)$$

(solve in Mellin space: Korchemsky/Marchesini 92, momentum space: Becher/Neubert 06)

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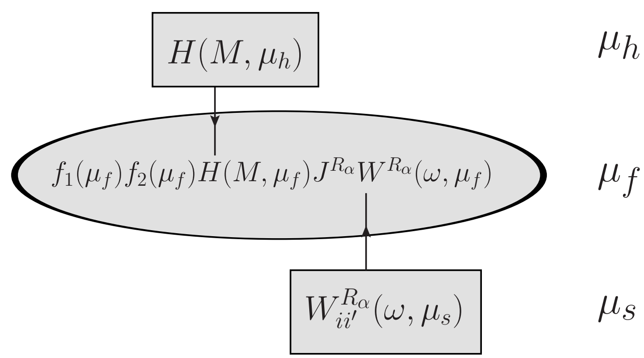
Resummation:

- evolve hard function from $\mu_h \sim 2M$ to μ_f
- evolve soft function from μ_s to μ_f



Ingredients for resummation at given accuracy

$$\hat{\sigma}_{pp'} \quad \propto \quad \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \ldots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \left\{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \ldots\right\} :$$

$$\left.\begin{array}{r}\text{NLL: tree-level}\\ \text{NNLL: one-loop}\end{array}\right\} H_i W_i^{R_\alpha};\ \left.\begin{array}{c}\text{one-loop}\\ \text{two-loop}\end{array}\right\} \gamma_s^r,\ \gamma_s^{H,R_\alpha}\ ;\ \left.\begin{array}{c}\text{two-loop}\\ \text{three-loop}\end{array}\right\} \gamma_{\text{cusp}}$$

(γ_s^r, γ_{cusp} known up to three loops: Moch, Vermaseren, Vogt 04/05)

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Soft anomalous dimension (Beneke, Falgari, CS 09; Czakon, Mitov, Sterman 09)

$$\gamma_{H,s}^{R_\alpha} = \frac{\alpha_s}{4\pi}(-2C_{R_\alpha}) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{R_\alpha}\left[-C_A\left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{40}{18}n_f\right] + \mathcal{O}(\alpha_s^3).$$

(extracted from Becher/Neubert 09, Korchemsky/Radyushkin 92, Kidonakis 09)

Coulomb resummation: (N)LO Coulomb-Green function

(Fadin, Khoze 87; Beneke, Signer, Smirnov 99,...)

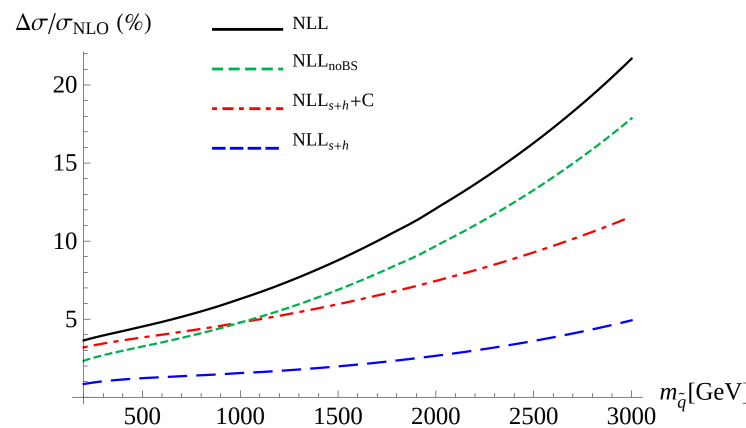
Squark -antisquarks at LHC

- Two production channels:

  $q_i\bar{q}_j \to \tilde{q}_k\overline{\tilde{q}_l} \quad , \qquad gg \to \tilde{q}_k\overline{\tilde{q}_l}$

- equal squark masses, no stop
- Matching to NLO result

  (Beenakker et.al. 96, PROSPINO)



($\sqrt{s} = 14$ TeV, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$MSTW08NLO)

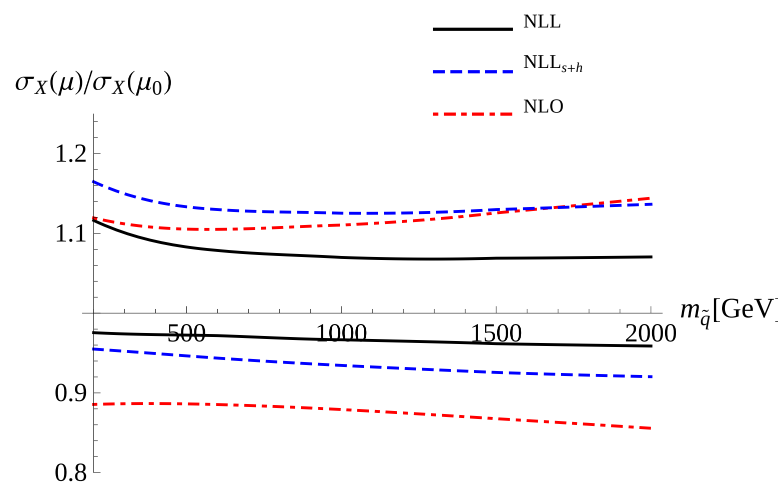
Resummed Results:

NLL: full Coulomb $\otimes$ res. soft

noBS: NLL w.o. bound states

$\rm NLL_{s+h}$: resummation of H and W

C: Coulomb resummation



($m_{\tilde{q}} = 1$ TeV, $\mu_i^0/2 < \mu_i < 2\mu_i^0$)

Reduced scale dependence

for combined NLL resummation

Good agreement of $\rm NLL_s$+C

with Mellin-approach

(Kulesza/Motyka; Beenakker et.al. 09)

$t\bar{t}$ production at Tevatron and LHC (Beneke, Falgari, Klein, CS, preliminary)

$\sigma_{t\bar{t}}$(pb)	Tevatron	LHC7	LHC10	LHC14
NLO	$6.50^{+0.32+0.33}_{-0.70-0.24}$	150^{+18+8}_{-19-8}	380^{+44+17}_{-46-17}	842^{+97+30}_{-97-32}
NNLO$_{\text{approx}}$	$7.13^{+0.00+0.36}_{-0.33-0.26}$	162^{+3+9}_{-3-9}	407^{+11+17}_{-5-18}	895^{+29+31}_{-7-33}
NNLO$_{\text{approx}}$ + NNLL	$7.14^{+0.13+0.36}_{-0.19-0.26}$	162^{+4+9}_{-2-9}	407^{+14+17}_{-4-18}	896^{+36+31}_{-7-33}
NLO + NNLL (Ahrens et.al. 10)	$6.48^{+0.17+0.32}_{-0.21-0.25}$	146^{+7+8}_{-7-8}	368^{+20+19}_{-14-15}	813^{+50+30}_{-36-35}

($m_t = 173.1$ GeV, $\mu_f = mt$, MSTW08NNLO PDF)

NLO corrections (Nason, Dawson Ellis 88, Beenakker et.al. 89/91)

(one-loop H_i for singlet/octet channels: Czakon/Mitov 08)

NNLO$_{\text{approx}}$ (also inplemented in `HATHOR`, Aliev et.al. 10)

$$\sigma^{(2)}_{q\bar{q}} = \frac{3.61}{\beta^2} + \frac{1}{\beta}\Big(- 140.4\ln\beta^2 + 32.1\ln\beta + 3.95\Big) + 910.2\ln\beta^4 - 1315.5\ln\beta^3 + 592.3\ln\beta^2 + 528.6\ln\beta + C^{(2)}_{qq}$$

- α_s^2 expansion of NNLL (Moch, Uwer (Langenfeld) 08/09)

+ all potential corrections (Beneke, Czakon, Falgari, Mitov CS, 09)

EFT approach to $\log\beta$, β^{-n} resummation for σ_{tot}

- use SCET+NRQCD to factorize soft and Coulomb gluons
- $\log\beta$ resummation from momentum space solution to RGEs

Colour structure of soft function

- diagonal basis to all orders for arbitrary colour
- two-loop soft anomalous dimension

Application to squark-antisquark production

- combined Soft and Coulomb resummation
- total corrections $4-10\%$ for $m_{\tilde{q}}=300$ GeV-2 TeV

Threshold expansion to $\mathcal{O}(\alpha_s^2)$ of $t\bar{t}$ cross section

NNLL resummation for $t\bar{t}$

- dominant higher-order corrections included in NNLO$_{\text{approx}}$
- discrepancy to NNLL from integrated $\frac{d\sigma}{dM_{tt}^2}$? (Ahrens et.al. 10)

Soft-collinear Lagrangian (Bauer et.al. 2000, Beneke et.al. 2002)

for quarks with momentum $p \sim n$, $n^2 = 0$: $(n \cdot \bar{n} = 2, n \cdot p_\perp = 0)$

$$\mathcal{L}_c = \bar{\xi}_c \left(in \cdot D + i\not{D}_{\perp c} \frac{1}{i\bar{n} \cdot D_c} i\not{D}_{\perp c} \right) \frac{\not{\bar{n}}}{2} \xi_c$$

Coupling to soft and collinear gluons:

$$iD_c = i\partial + gA_c \qquad iD = iD_c + gA_s$$

Field redefinition (Bauer, Pirjol, Stewart 01)

$$\xi_c(x) = S_n(x_-)\xi_c^{(0)}(x) \qquad S_n(x) = \mathrm{P}\exp\left[ig_s \int_{-\infty}^{0} dt\, n \cdot A_s^a(x+nt)T^a\right]$$

Wilson line satisfies

$$n \cdot \partial S_n(x_-) = ig_s n \cdot A_s^a(x_-)T^a S_n(x_-)$$

$\Rightarrow$ soft gluons decouple in Lagrangian:

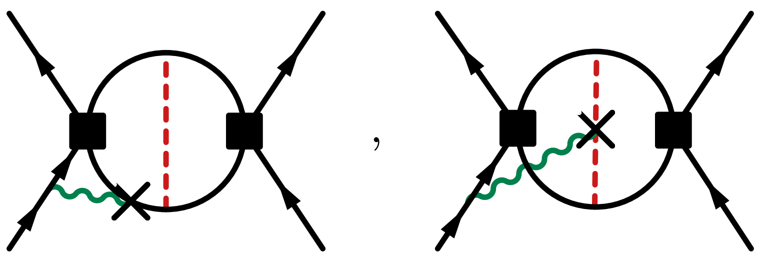
$$\bar{\xi}_n(in \cdot D_s)\xi_n = \bar{\xi}_n^{(0)}(in \cdot \partial)\xi_n^{(0)}$$

Subleading PNRQCD and SCET interactions:

$$\psi^\dagger \vec{x} \cdot \vec{E}_{us}(x_0, 0)\psi'^\dagger , \quad \bar{\xi}\left(x_\perp^\mu n_-^\nu\, W_c\, g F_{\mu\nu}^{\rm us} W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi \ldots$$

Soft gluons not decoupled by field redefinitions.

Possibly relevant at NNLL in soft $\otimes$ potential corrections :



$$\sim \frac{\alpha_s}{\beta} \times \alpha_s \beta \log\beta \sim \alpha_s^2 \log\beta$$

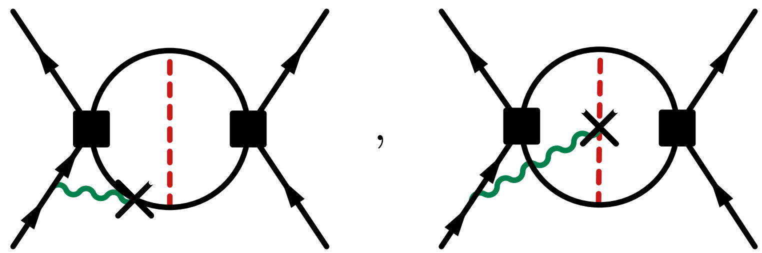
Related to three-parton colour correlations in IR singularities of amplitudes (Ferroglia et.al. 09)

$\sigma_{\rm tot}$: effects vanish at NNLL! (Beneke, Czakon,Falgari, Mitov, CS 09)

- no collinear/potential correction $\sim \beta$ for $k_\perp = 0$
- no potential/soft corrections due to rotational invariance

  (no heavy particle three-momentum available)

Subleading soft $\otimes$ potential corrections:



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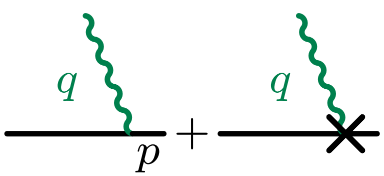
Expansion of Hg_sH vertex: $\left(p = Mv + r,\ (r^0, \vec{r}) \sim (\lambda, \sqrt{\lambda}),\ q \sim \lambda\right)$:



$$: \quad \frac{v^\mu}{r^0 + q^0 - \frac{\vec{r}^2}{2M}} + \left[\frac{\vec{r}/M}{r^0 + q^0 - \frac{\vec{r}^2}{2M}} - v^\mu \, \frac{\vec{r}\cdot\vec{q}}{M} \left(\frac{1}{r^0 + q^0 - \frac{\vec{r}^2}{2M}} \right)^2 \right] + \mathcal{O}(\beta^2)$$

$\Rightarrow$ Integrals of the form ($\vec{v} = 0$ in partonic CMS, no $\vec{q}$ in denominator)

$$\int d^D q \prod_i d^D r_i \, \frac{\{(\vec{k}_- \cdot \vec{r}_i)\ ,\ (\vec{q} \cdot \vec{r}_i)\}}{F(q^0, (k_- \cdot q), r_i^0, (\vec{r}_i + \vec{r}_j)^2)} = 0$$

Vanish since no external potential 3-momentum available!

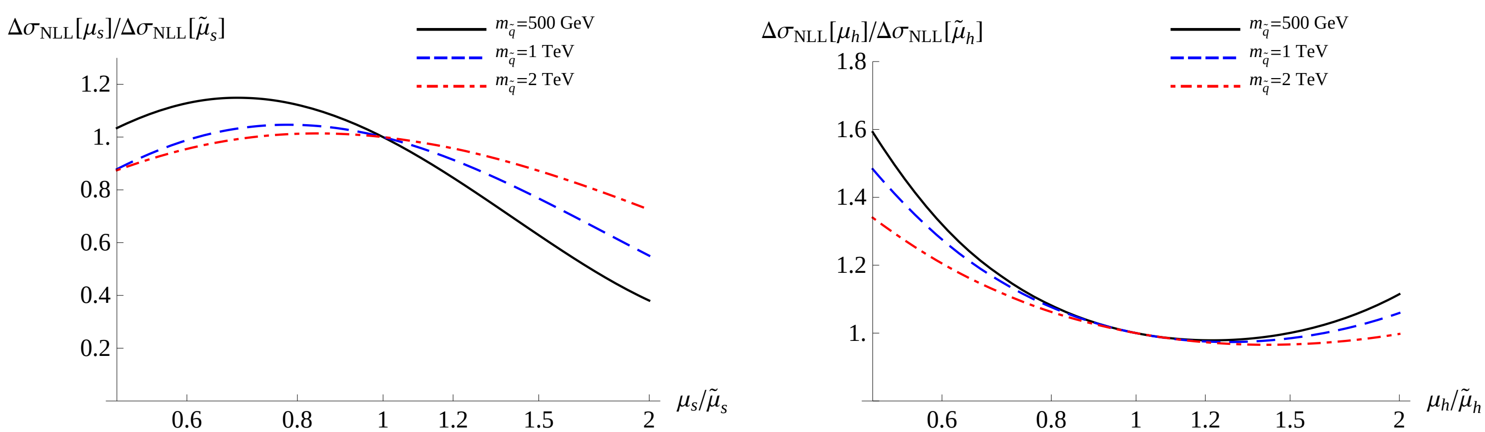
Choice of scales for resummation in momentum space

Soft scale $\tilde{\mu}_s$ that minimizes hadronic $\Delta\sigma^{\text{NLO}}_{\text{soft}}$ (Becher, Neubert, Xu 07)

$$\tilde{\mu}_s/m_{\tilde{q}} \approx 0.5\ldots 0.2 \qquad \text{for } m_{\tilde{q}} = 0.5,\ldots 2 \text{ TeV}$$

Hard scale: $\tilde{\mu}_h = 2m_{\tilde{q}}$

Dependence on scale choices:



($\sqrt{s} = 14$ TeV, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$)

Coulomb scale: $\mu_C = \max\{2m_{\tilde{q}}\beta, C_F m_{\tilde{q}}\alpha_s(\mu_C)\}$

Comparison to Mellin-approach: (Kulesza, Motyka 08/09, Beenakker et.al. 09)

Good agreement for appropriate choice of scales ($\mu_h = \mu_f$: NLL$_s$):

$m_{\tilde{q}}$[GeV]	NLO[pb]	NLL$_{\text{Mellin}}$[pb]	NLL$_s$[pb]	NLL [pb]
200	1.3×10^3	1.31×10^3 (1%)	1.31×10^3 (1%)	1.33×10^3 (2%)
500	1.6×10^1	1.61×10^1 (1.2%)	1.62×10^1 (1.3%)	1.65×10^1 (3%)
1000	2.89×10^{-1}	$2.93 \times 10^{-1}(1.7\%)$	$2.94 \times 10^{-1}(1.7\%)$	$3.02 \times 10^{-1}(4.4\%)$
2000	1.11×10^{-3}	$1.14 \times 10^{-3}(3.4\%)$	$1.14 \times 10^{-3}(3.1\%)$	1.21×10^{-3} (9%)
3000	7.13×10^{-6}	$7.59 \times 10^{-6}(6.4)\%$	7.54×10^{-6}	8.34×10^{-6}

(LHC 14 TeV, $m_{\tilde{g}} = m_{\tilde{q}}$)

Top-pair production at NNLL

All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms (Beneke, Czakon, Falgari, Mitov, CS 09)

Pure soft corrections:

$$\Delta\sigma_{\mathsf{s}}^{(2)} \sim \alpha_s^2 (c_{\mathsf{LL}}^{(2)} \ln^4 \beta + c_{\mathsf{NLL}}^{(2)} \ln^3 \beta + c_{\mathsf{NNLL,2}}^{(2)} \ln^2 \beta + \underbrace{c_{\mathsf{NNLL,1}}^{(2)} \ln \beta}_{\text{2-loop } \gamma_{H,s}})$$

Potential corrections: 2nd Coulomb, NLO potentials

$$\Delta\sigma_{\mathsf{p}}^{(2)} \sim \alpha_s^2 \left(\frac{c_{C^2}}{\beta^2} + \frac{1}{\beta}(c_{\mathsf{C,0}}^{(2)} + c_{\mathsf{C,1}}^{(2)} \log \beta) + \underbrace{c_{\mathsf{n\text{-}C}}^{(2)} \ln \beta}_{\text{spin-dependent}} \right)$$

(extracted from Beneke, Signer, Smirnov 99, Czarnecki/Melnikov 97/01)

mixed Coulomb/soft, hard corrections:

$$\Delta\sigma_{\mathsf{p}\otimes\,\mathsf{sh}}^{(2)} \sim\sim \frac{\alpha_s}{\beta} \alpha_s (c_{\mathsf{LL}}^{(1)} \ln \beta^2 + c_{\mathsf{NLL}}^{(1)} \ln \beta + c + \underbrace{H^{(1)}}_{\text{process dependent}})$$

Potential corrections:

- 2nd Coulomb correction
- NLO Coulomb potentials:

$$\tilde{V}_{\mathrm{C}}^{(1)}(\boldsymbol{p},\boldsymbol{q}) = \frac{D_{R_\alpha}\alpha_s^2}{\boldsymbol{q}^2}\left(a_1 - \beta_0 \ln\frac{\boldsymbol{q}^2}{\mu^2}\right)$$

- Non-Coulomb potential:

$$\tilde{V}_{\mathrm{nC}}^{(1)}(\boldsymbol{p},\boldsymbol{q}) = \frac{4\pi D_{R_\alpha}\alpha_s}{\boldsymbol{q}^2}\left[\frac{\pi\alpha_s|\boldsymbol{q}|}{4m}\left(\frac{D_{R_\alpha}}{2}+C_A\right)+\frac{\boldsymbol{p}^2}{m^2}+\frac{\boldsymbol{q}^2}{m^2}\,v_{\mathrm{spin}}\right],$$

($v_{\mathrm{spin}} = 0$ (singlet); $-2/3$ (triplet))

Corrections to cross section:

$$\Delta\hat{\sigma}_{\mathrm{nC}} = \hat{\sigma}^{(0)}\,\alpha_s^2\,\ln\beta\,\left[-2D_{R_\alpha}^2\,(1+v_{\mathrm{spin}}) + D_{R_\alpha}C_A\right]$$

(extracted from Beneke, Signer, Smirnov 99, Pineda, Signer 06)