Analytic computation of top quark pair production at hadron colliders

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TOP QUARKS AT THE LHC

- Top quarks play a central role since
  - heavy particles are the gate to new physics
  - produced in large quantities
- Analytic computations
  - come with good numerical stability and speed
  - are versatile adaptable (implement in Monte Carlos, improve numerical codes)
  - give insight into structure of amplitude → find new techniques
  - freely accessible

TOP QUARK PAIR STUDIES AT NLO

- $pp \to t\bar{t}$ well studied process:
  - NLO cross section [ELLIS,NASON,DAWSON]
  - tensorial amplitude (analyt.) [KÖRNER, MEREBASHVILI]
  - spin correlations [BERNREUTHER, BRANDENBURGER,SI,UWER]
  - generalized unitarity (num.) [MELNIKOV,SCHULZE]
- $pp \to t\bar{t} + j$: background for $t\bar{t}H$

  [DITTMAIER, UWER, WEINZIERL]

  [MELNIKOV, SCHULZE]

  [BEVILACQUA ET.AL. HELAC−1LOOP]
- $pp \to t\bar{t}b\bar{b}$ background for $t\bar{t}H$

  [BEVILACQUA ET.AL. HELAC−1LOOP]

  [BREDENSTEIN,DENNER,DITTMAIER,POZZORINI]
- $pp \to t\bar{t} + 2j$: background for new physics

  [BEVILACQUA ET.AL. HELAC−1LOOP]

Our approach

- uses helicity amplitudes aiming at compact representation of the result
- analytic implementation of generalized unitarity for massive particles
- allows further investigation of the structure of the amplitude especially the rational terms

SPINOR HELICITY FORMALISM

- results given in terms of 2 dim. Weyl spinors

$$|p\rangle = |p+\rangle \qquad |p] = |p-\rangle$$

$$2\,[\,p_1\,|\gamma^\mu\,|\,q_2\rangle\langle q_1\,|\gamma^\mu\,|\,p_2\,] = \langle p_1\,p_2\rangle\,[\,q_1\,q_2\,]$$

- massless momenta and polarisation vectors

$$2q^\mu = \langle q\,|\gamma^\mu\,|\,q\,] \qquad \epsilon_+^\mu(p,q) = \frac{\langle q\,|\gamma^\mu\,|\,p\,]}{\sqrt{2}\langle q\,p\rangle} \qquad \epsilon_-^\mu(p,q) = \frac{\langle p\,|\gamma^\mu\,|\,q\,]}{\sqrt{2}[\,p\,q\,]}$$

$$\langle p\,q\rangle \sim e^{i\phi_{pq}}\,\sqrt{pq} \qquad [\,p\,q\,] \sim e^{-i\phi_{pq}}\,\sqrt{pq}$$

- symmetry reduces number of independent amplitudes

HELICITY FORMALISM - HEAVY QUARKS

- helicity states of spinors defined w.r.t. a massless vector [KLEISS, STIRLING]

$$u_{\pm}\left(Q(q,\eta);\eta\right) = \frac{(\not{Q} + m)\,|\eta\mp\rangle}{\langle q\pm,\eta\mp\rangle}$$

- projection of the momentum onto the light cone

$$q^{\mu} = Q^{\mu} - \frac{m^2}{2Q\eta}\eta^{\mu}$$

- keeping η arbitrary yields:
  - enlarged number of independent spinor products
  - reduced number of independent helicity amplitudes

$$u_{-}(Q(q,\eta);q,\eta) = \frac{\langle q\,\eta\rangle}{m}u_{+}(Q(q,\eta);\eta,q)$$

COMPACT FORM FOR TREE AMPLITUDES

$$A_4(1_t^+,2^+,3^+,4_{\bar{t}}^+) = -im^3\frac{\langle\eta_1\,\eta_4\rangle[2\,3]}{\langle 2\,3\rangle\langle 2|1|2]}$$

$$A_4(1_t^+,2^+,3^-,4_{\bar{t}}^+) = -im\frac{\langle 3|1|2]\left(\langle\eta_1\,\eta_4\rangle\langle 3|1|2] + \langle\eta_1\,3\rangle\langle\eta_4\,3\rangle[2\,3]\right)}{s_{23}\langle 2|1|2]}$$

$$A_4(1_t^+,2^+,4_{\bar{t}}^+,3^+) = im^3\frac{\langle\eta_1\,\eta_4\rangle[2\,3]^2}{\langle 2|1|2]\langle 3|1|3]}$$

$$A_4(1_t^+,2^-,4_{\bar{t}}^+,3^+) = -im\frac{\langle 2|1|3]\left(\langle\eta_1\,\eta_4\rangle\langle 2|1|3] + \langle 2\,\eta_1\rangle\langle 2\,\eta_4\rangle[2\,3]\right)}{\langle 2|1|2]\langle 3|1|3]}$$

$$A_4(1_t^+,2_q^-,3_{\bar{q}}^+,4_{\bar{t}}^+) = m\frac{\langle\eta_1 2\rangle\langle\eta_4|4|3] + \langle\eta_4 2\rangle\langle\eta_1|1|3]}{s_{14}}$$

$$\mathcal{A}_4^{(0)}(1_t,2,3,4_{\bar{t}}) = \sum_{P(2,3)} [T^{a_2}T^{a_3}]_{j_1}^{i_4} A_4^{(0)}(1_t,2,3,4_{\bar{t}})$$

$$\mathcal{A}_4^{(1)}(1_t,2,3,4_{\bar{t}}) = \sum_{P(2,3)} N[T^{a_2}T^{a_3}]_{j_1}^{i_4} A_{4;1}^{(1)}(1_t,2,3,4_{\bar{t}}) + \delta_{a_2}^{a_3}\delta_{j_1}^{i_4} A_{4;3}^{(1)}(1_t,4_{\bar{t}};2,3).$$

with

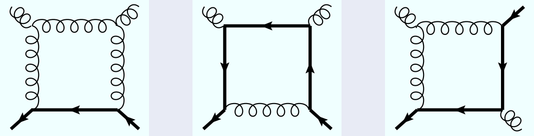
$$A_{4;1}^{(1)}(1_t,2,3,4_{\bar{t}}) = A^{[L]}(1_t,2,3,4_{\bar{t}}) - \frac{1}{N^2}A^{[R]}(1_t,2,3,4_{\bar{t}})$$

$$A_{4;3}^{(1)}(1_t,4_{\bar{t}};2,3) = \sum_{P(2,3)} \left\{A^{[L]}(1_t,2,3,4_{\bar{t}}) + A^{[L]}(1_t,2,4_{\bar{t}},3) + A^{[R]}(1_t,2,3,4_{\bar{t}})\right\}$$

[CVITANOVIC;BERN,DIXON,KOSOWER]

COLOR STRUCTURE IN THE GLUON CHANNEL

$$\mathcal{A}_4^{(1)}(1_t, 2, 3, 4_{\bar{t}}) = \sum_{P(2,3)} N[T^{a_2} T^{a_3}]_{j_1}^{i_4} A_{4;1}^{(1)}(1_t, 2, 3, 4_{\bar{t}}) + \delta_{a_2}^{a_3} \delta_{j_1}^{i_4} A_{4;3}^{(1)}(1_t, 4_{\bar{t}}; 2, 3).$$



$$A_{4;1}^{(1)}(1_t, 2, 3, 4_{\bar{t}}) = A^{[L]}(1_t, 2, 3, 4_{\bar{t}}) - \frac{1}{N^2} A^{[R]}(1_t, 2, 3, 4_{\bar{t}})$$

$$A_{4;3}^{(1)}(1_t, 4_{\bar{t}}; 2, 3) = \sum_{P(2,3)} \left\{ A^{[L]}(1_t, 2, 3, 4_{\bar{t}}) + A^{[L]}(1_t, 2, 4_{\bar{t}}, 3) + A^{[R]}(1_t, 2, 3, 4_{\bar{t}}) \right\}$$

COLOR STRUCTURE IN THE QUARK CHANNEL

$$\mathcal{A}_4^{(0)}\left(1_t, 2_q, 3_{\bar{q}} 4_{\bar{t}},\right) = \left(\delta_{j_1}^{i_3}\delta_{j_2}^{i_4} - \frac{1}{N}\delta_{j_1}^{i_4}\delta_{j_2}^{i_3}\right) A_4^{(0)}\left(1_t, 2_q, 3_{\bar{q}}, 4_{\bar{t}}\right)$$

$$\mathcal{A}_4^{(1)}\left(1_t, 2, 3, 4_{\bar{t}}\right) = N\,\delta_{j_1}^{i_3}\delta_{j_2}^{i_4}\, A_{4;1}^{(1)}\left(1_t, 2, 3, 4_{\bar{t}}\right) + \delta_{j_1}^{i_4}\delta_{j_2}^{i_3}\, A_{4;2}^{(1)}\left(1_t, 2, 3, 4_{\bar{t}}\right)$$

with

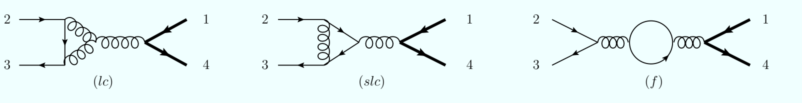
$$A_{4;1}^{(1)}\left(1_t, 2, 3, 4_{\bar{t}}\right) = A_4^{lc}\left(1_t, 2, 3, 4_{\bar{t}}\right) - \frac{2}{N^2}\left(A_4^{lc}\left(1_t, 2, 3, 4_{\bar{t}}\right) + A_4^{lc}\left(1_t, 3, 2, 4_{\bar{t}}\right)\right) - \frac{1}{N^2} A_4^{slc}\left(1_t, 2, 3, 4_{\bar{t}}\right)$$

and

$$A_{4;2}^{(1)}\left(1_t, 2, 3, 4_{\bar{t}}\right) = A_4^{lc}\left(1_t, 3, 2, 4_{\bar{t}}\right) + \frac{1}{N^2}\left(A_4^{lc}\left(1_t, 3, 2, 4_{\bar{t}}\right) + A_4^{lc}\left(1_t, 2, 3, 4_{\bar{t}}\right)\right) + \frac{1}{N^2} A_4^{slc}\left(1_t, 3, 2, 4_{\bar{t}}\right)$$

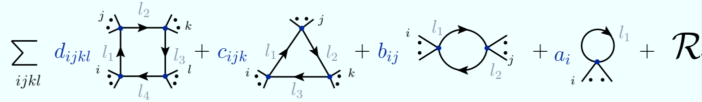
COLOR STRUCTURE IN THE QUARK CHANNEL

$$\mathcal{A}_4^{(1)}(1_t,2,3,4_{\bar{t}}) = N\,\delta_{j_1}^{i_3}\delta_{j_2}^{i_4}\,A_{4;1}^{(1)}(1_t,2,3,4_{\bar{t}}) + \delta_{j_1}^{i_4}\delta_{j_2}^{i_3}\,A_{4;2}^{(1)}(1_t,2,3,4_{\bar{t}})$$

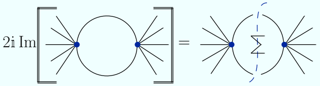


$$A_{4;1}^{(1)}(1_t,2,3,4_{\bar{t}}) = A_4^{lc}(1_t,2,3,4_{\bar{t}}) - \frac{2}{N^2}\left(A_4^{lc}(1_t,2,3,4_{\bar{t}}) + A_4^{lc}(1_t,3,2,4_{\bar{t}})\right) - \frac{1}{N^2}A_4^{slc}(1_t,2,3,4_{\bar{t}})$$

ONE LOOP AMPLITUDES IN GAUGE THEORY



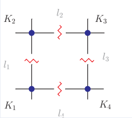
- in 4 dim. box is highest possible topology [PASSARINO, VELTMAN; MELROSE]

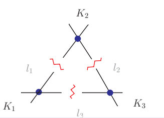


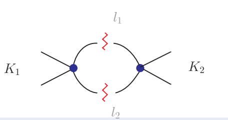
- optical theorem relates product of tree amplitudes to imaginary part of one loop amplitude
- imaginary parts can be uniquely attributed to the scalar integrals
- applying Cutkosky rules at the amplitude level allows to construct amplitude at the integrand level [BERN, DIXON, KOSOWER]

GENERALIZED UNITARITY

- interpretation of cuts as kinemtical contraints on the loop momenta $\Rightarrow$ upto 4 cuts in 4 dimensions
- full benefit from the integral basis since directly aiming at a specific coefficient $\Rightarrow$ # cuts $\approx$ topology





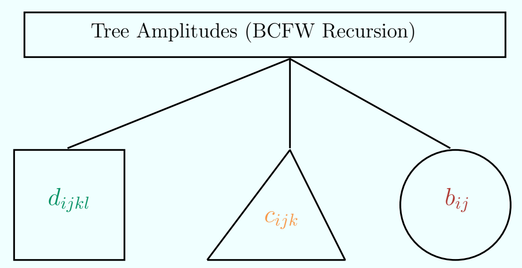


$$d_{ijkk} = \prod_{i=1}^{4} A_i^{(0)} \qquad c_{ijk} = \lim_{t\to\infty} \prod_{i=1}^{3} A_i^{(0)}(t)|_{t^0} \qquad b_{ij} = \lim_{t,y\to\infty} \prod_{i=1}^{2} A_i(t,y)$$

[Britto,Cachazo,Feng;Ossala,Papadopoulos,Pittau;Forde;Killgore;Mastrolia;Bern,Dixon,Dunbar,Kosower]

Strategy of the calculation

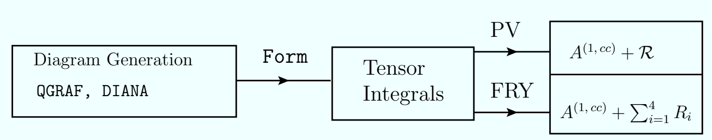
- generalized untitarity for integral coefficients



- improved by constraints from UV-IR singularities
- semi automatized `Form – Maple` code

Strategy of the calculation

- tensor reduction for rational and mass renormalization part



- necessary since:
  - on-shell wave-function bubbles are cut-divergent
  - mass renormalization is required to obtain gauge invariant result
- allow for independent crosscheck

numerical tensor reduction

- tensor reduction using higher dimensional integrals [DAVYDYCHEV]
- reduce to standard scalar integrals by shift identity

$$I_n^{D+2} = \frac{A}{G} I_n^D + \sum_k \frac{B_k}{G} I_{n-1;k}^D$$

[BERN,DIXON,KOSOWER;FLEISCHER,JEGERLEHNER,TARASOV,]

- ellimination of Gram determinant G during step above

[FLEISCHER,RIEMANN;DIAKONIDIS,FLEISCHER,RIEMANN,TAUSK]

- direct match with D dimensional cut basis
- full numerical implementation in C++ [YUNDIN (IN PREPARATION)]
- analytic implementation for rational terms in $pp \to t\bar{t}$

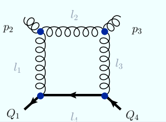
EXAMPLE: A BOX COEFFICIENT

$$l^2 = 0 \qquad 2(l, p_3) = 0$$

$$2(l, p_2) = 0 \qquad 2(l, Q_1) = -m^2 - 2(p_2, Q_1)$$

$$2\, l^\mu = \alpha_1 \langle p_2 | \gamma^\mu | p_2] + \alpha_2 \langle p_3 | \gamma^\mu | p_3] + \alpha_3 \langle p_2 | \gamma^\mu | p_3] + \alpha_4 \langle p_3 | \gamma^\mu | p_2]$$

$$2\, l^\mu = \frac{\langle p_3 | Q_4 | p_3]}{\langle p_2 | Q_4 | p_3]} \langle p_2 | \gamma^\mu | p_3] \qquad 2\, (l^\mu)^* = \frac{\langle p_3 | Q_4 | p_3]}{\langle p_3 | Q_4 | p_2]} \langle p_3 | \gamma^\mu | p_2]$$



$$c_4^{[L]} \left(1_t^+, 2_g^+, 3_g^+, 4_{\bar{t}}^+ \right) = m^3 \frac{\langle \eta_1\, \eta_4 \rangle [\, p_3\, p_2\,]^2}{\langle \eta_1\, q_1 \rangle \langle q_4\, \eta_4 \rangle}$$

[BADGER]

UV-IR pole structure

- universal singular behaviour of massless and massive one-loop amplitudes well studied subject [Catani,Dittmaier,Tróscányi;Mitov,Moch]
- usually used as check only
- can be cast into constraints for integral coefficients
  $\Rightarrow$ become constructive element of calculation [Badger]
- massless legs (soft and collinear divergencies)

$$\frac{2\,c_4^{[L]}}{s_{23}\,\langle 2|1|2]} + \frac{c_{3|1,2}^{[L]} + c_{3|3,4}^{[L]}}{2\,\langle 2|1|2]} + \frac{c_{3|2,3}^{[L]}}{2\,s_{23}} = -\,2\,A_4^{(0)}\,(1_t, 2, 3, 4_{\bar{t}})$$

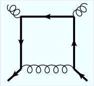
$$\frac{2\,c_4^{[lc]}}{s_{23}\,\langle 2|1|2]} + \frac{c_{3|1,2}^{[lc]} + c_{3|3,4}^{[lc]}}{2\,\langle 2|1|2]} + \frac{c_{3|2,3}^{[lc]}}{2\,s_{23}} = -\,A_4^{(0)}\,(1_t, 2, 3, 4_{\bar{t}})$$

$$\frac{c_{3|1,4}^{[slc]}}{2\,s_{23}} = A_4^{(0)}\,(1_t, 2, 3, 4_{\bar{t}})$$

BOX AND TRIANGLE COEFFICIENTS

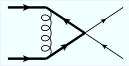
- massive external legs (soft and collinear divergencies)

$$\frac{2\, c_4^{[R]}}{(s_{23} - 2m^2)\langle 2|1|2]} + \frac{c_{3|2,3}^{[R]}}{s_{23} - 2m^2} = -A_4^{(0)}(1_t, 2, 3, 4_{\bar{t}})$$



$\Rightarrow$ elliminates square root + three mass triangle

$$\frac{c_{3|2,3}^{[s/c]}}{s_{23} - 2m^2} = A_4^{(0)}(1_t, 2, 3, 4_{\bar{t}})$$



BUBBLES AND TADPOLES

- further changes to the integral base:

$$
\begin{aligned}
I_2(s_{23},0,0;m^2) &= I_2(s_{23},0,0) - I_2(m^2,0,m^2) + 2 \\
F_2(s_{12},0,m^2) &= I_2(s_{12},0,m^2) - I_2(m^2,0,m^2) \\
F_2(s_{23},m^2,m^2) &= I_2(s_{23},m^2,m^2) - I_2(m^2,0,m^2) \\
I_1(m^2) &= m^2 \left(I_2(m^2,0,m^2) - 1 \right)
\end{aligned}
$$

- collects all $\log(m^2)$ dependence in one place $\Rightarrow$ fixed by $\frac{1}{\epsilon}$ poles
  - bypass the problem of cut divergencies for the coefficients ...
  - temporarily removes the need to calculate the tadpole coefficient ...
  - ...but shifts this issues into the rational terms

all plus result in the gluon channel

$$A_4^{[L]}(1_t^+, 2_g^+, 3_g^+, 4_{\bar{t}}^+) =$$

$$+ I_4(s_{12}, s_{23}, 0, m^2, m^2, 0, 0, 0, m^2, 0)(-\langle\eta_1\eta_4\rangle[23]^2 m^3)$$

$$+ F_2(s_{12}, 0, m^2)(\frac{2\langle\eta_1\eta_4\rangle[23]m^3}{\langle 23\rangle\langle 2|1|2]} + \frac{2\langle\eta_1\eta_4\rangle[23]m^5}{\langle 23\rangle\langle 2|1|2]^2} - \frac{[23]\langle\eta_1|K_{12}|K_{23}|\eta_4\rangle m^3}{\langle 23\rangle\langle 2|1|2]^2})$$

$$+ (I_2(m^2, 0, m^2) - 2)(-\frac{\langle\eta_1\eta_4\rangle[23]m^3}{2\langle 23\rangle\langle 2|1|2]})$$

$$- \frac{(\langle\eta_1\eta_4\rangle\langle 2|1|2] - \langle\eta_1 2\rangle\langle\eta_4 3\rangle[23])m}{3\langle 23\rangle^2} + \frac{[23](\langle\eta_1\eta_4\rangle\langle 2|1|2] + \langle\eta_1|K_{12}|K_{23}|\eta_4\rangle)m}{2\langle 23\rangle\langle 2|1|2]}$$

$$A^{[f]}\left(1_t^+, 2^+, 3^+, 4_{\bar{t}}^+\right) = -2\, m\, m_f^2\, \frac{\langle 2\,\eta_1\rangle\langle\eta_4\,|\,4\,|\,2\,] + \langle 2\,\eta_4\rangle\langle\eta_1\,|\,1\,|\,2\,]}{\langle 2\,3\rangle^2 s_{23}} \times$$

$$\left(2F_2(s_{23}, m_f^2, m_f^2) + s_{23}\, I_3(0, 0, s_{23}, m_f^2, m_f^2, m_f^2)\right)$$

$$- m\, \frac{\langle 2\,\eta_1\rangle\langle\eta_4\,|\,4\,|\,2\,] + \langle 2\,\eta_4\rangle\langle\eta_1\,|\,1\,|\,2\,]}{\langle 2\,3\rangle^2 s_{23}} \times \left(8\, m_f^2 + \frac{1}{3}\, s_{23}\right)$$

SUMMARY

- successfull application of analytic on-shell techniques to massive case
- automatized generalized unitarity for arbitrary masses possible
- derived compact expressions for $pp \to t\bar{t}$ [BADGER,RS,YUNDIN (TO BE APPEAR)]
- phenomenological application by implementation into open source NLO codes [MCFM:CAMPBELL,ELLIS;POWHEG-BOX:AIOLI,NASON,OLEARI,RE]
- further investigations of structure of rational terms necessary