Gravitational shock wave collision and matter equilibration in heavy ion collisions

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Outline

- Gravitational shock wave collision as a model of HIC
- Trapped surface formation from collision of gravitational shock waves with point source
- Trapped surface formation from collision of gravitational shock waves with wall source
- NLO stress tensor in wall shock wave collision and origin of critical impact parameter

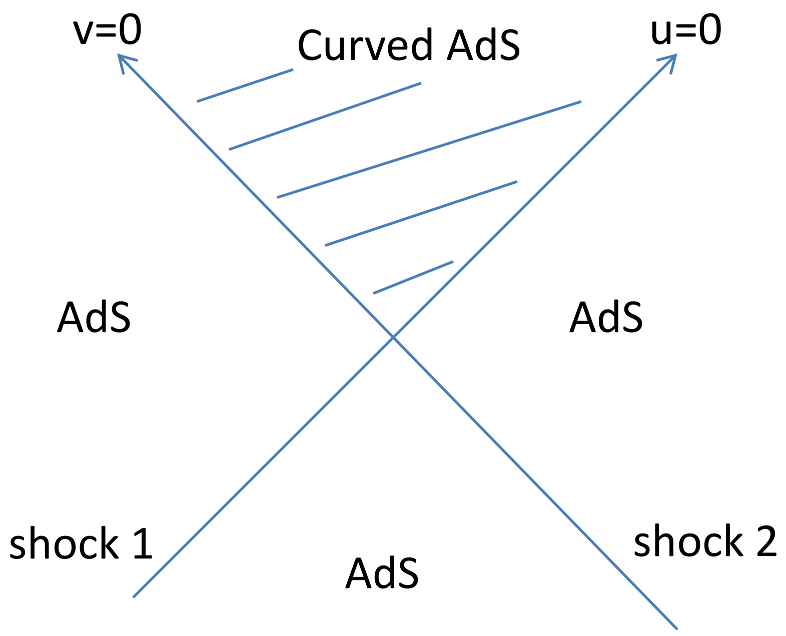
Equilibration of matter in HIC

Stationery AdS black hole ~ QGP matter in equilibrium

Equilibration of matter ~ formation of horizon

Apparent horizon(trapped surface) vs Event horizon

Gravitational shock wave collision



$$u = t - x_{//}$$

$$v = t + x_{//}$$

Shock Waves in AdS

AdS background $$ds^2 = L^2 \frac{-dudv + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2}$$

Shock Wave in AdS $$ds^2 = L^2 \frac{-dudv + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2} + L \frac{\Phi(x^1, x^2, z)}{z} \delta(u) du^2$$

$$\left(\Box - \frac{3}{L^2} \right) \Phi = 16\pi G_5 J_{uu}$$

J_{uu}: 5D source in the bulk

$\Box$: Laplacian in space H_3

$$ds^2_{H3} = L^2 \frac{(dx^1)^2 + (dx^2)^2 + dz^2}{z^2}$$

Gubser et al, **arXiv:0805.1551** [hep-th]

Nucleus modelled by shock wave

$$J_{uu} = E\delta(z-L)\delta^{(2)}(x_{\perp})$$

$$\left(\Box - \frac{3}{L^2}\right)\Phi = 16\pi G_5 J_{uu}$$

$$ds^2 = L^2\frac{-du dv + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2} + L\frac{\Phi(x^1, x^2, z)}{z}\delta(u)du^2$$

corresponding stress energy tensor: $T_{uu} = \dfrac{2L^4 E}{\pi\left(L^2 + (x^1)^2 + (x^2)^2\right)^3}\delta(u)$

L: characteristic size of the nucleus
E: energy of colliding nucleus

Tune L, E properly, use point shock wave to mimic relativistic nucleus

Gubser et al, **arXiv:0805.1551** [hep-th]

Condition of Trapped Surface

Shock Wave

$$ds^2 = L^2 \frac{-dudv + dx_\perp^2 + dz^2}{z^2} + \frac{\Phi(x_\perp, z)\delta(u)du^2}{z}$$

Trapped Surface at

$$u = 0, v = \Psi(x_\perp, z)$$

Condition of (marginally)Trapped Surface: vanishing of expansion

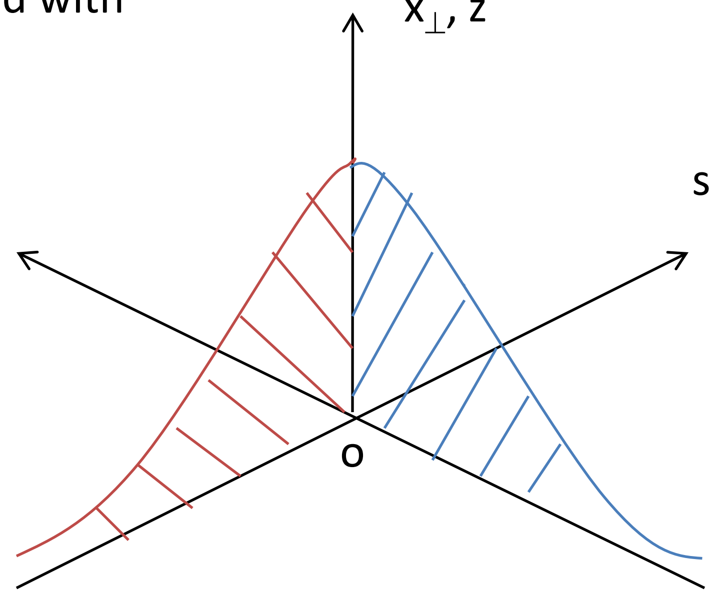
$$\theta \equiv h^{\mu\nu}\nabla_\mu l_\nu \leq 0$$

$h^{\mu\nu}$ induce metric

l_μ null vector normal to trapped surface

Merging of Two Trapped Surfaces

Merging of the trapped surfaces associated with two shocks



When trapped Surfaces meet at u=0, v=0:

Continuity of Trapped Surface: $\Psi_1(x_\perp,z)=v=0$, $\Psi_2(x_\perp,z)=u=0$

Continuity of null normal vector: $l_1=l_2$

Unusual Boundary Value Problem

Interior region:

$$\left(\Box - \frac{3}{L^2}\right)(\Psi_1 - \Phi_1) = 0$$

$$\left(\Box - \frac{3}{L^2}\right)(\Psi_2 - \Phi_2) = 0$$

On the boundary C:

$$\Psi_1|_{\mathcal{C}} = \Psi_2|_{\mathcal{C}} = 0$$

$$\nabla\Psi_1 \cdot \nabla\Psi_2|_{\mathcal{C}} = 4$$

Impact parameter: b

Minkowski
b=0: Penrose 1974
b≠0: Giddings and Eardley **gr-qc/0201034**
critical impact parameter

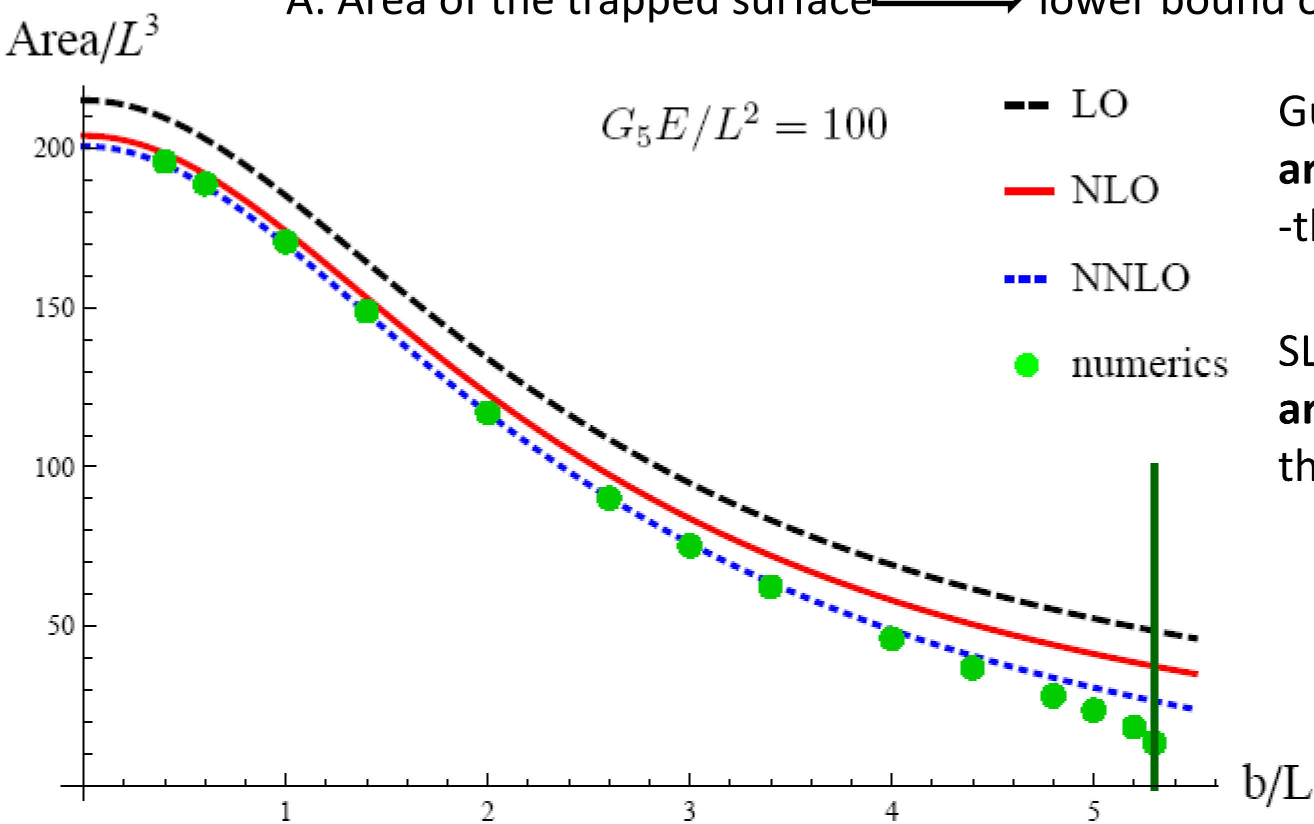
AdS
Gubser et al, **arXiv:0805.1551** [hep-th]

Entropy lower bound

$$S_{trapped} = \frac{2A}{4G_5} = \frac{1}{2G_5}\int \sqrt{g}d^3x$$

$$\frac{L^3}{G_5} = \frac{2N_c^2}{\pi}$$

A: Area of the trapped surface ⟹ lower bound of entropy production



Gubser et al **arXiv:0902.4062** [hep-th]

SL, E. Shuryak **arXiv:0902.1508** [hep-th]

First order transition

Limitation of Point shock waves

Nuclei is not a rigid body. A boost of nuclei will cause its parton density to grow and saturate at high energy.
Saturation scale should be incorporated in the model as the most important scale.

Nevertheless, the existence of ciritcal behavior is robust, insensitive to the nature of the source.

Shock wave for infinite nucleus

$$ds^2 = L^2 \frac{-du dv + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2} + L \frac{\Phi(x^1, x^2, z)}{z} \delta(u) du^2$$

$$\left(\Box - \frac{3}{L^2} \right) \Phi(z) = -16\pi \frac{G_5 E}{L^2} \delta(z - z_0)$$

$$\Phi(z) = \begin{cases} 4\pi G_5 E \frac{z^3}{z_0^4} & z < z_0 \\ 4\pi G_5 E \frac{1}{z} & z > z_0 \end{cases}$$

$1/z_0$: saturation scale

$$T_{uu} = \frac{E L^2}{z_0^4} \delta(u)$$

Trapped surface for collisions of infinite nucleus

Consider the collisions of two infinite shock waves, with the same energy density on the boundary

$$\mu = \frac{E_1 L^2}{z_1^4} = \frac{E_2 L^2}{z_2^4}$$

$$z^2 \Psi_i'' - z\Psi_i' - 3\Psi_i = -16\pi G_5 E_i \delta(z - z_i)$$
$$\Psi_i(z_a) = \Psi_i(z_b) = 0$$

Trapped surface: $z_a < z < z_b$

Entropy production per unit transverse area: $s = \frac{S}{\int d^2x} = \frac{N_c^2}{2\pi}\left(\frac{1}{z_a^2} - \frac{1}{z_b^2}\right)$

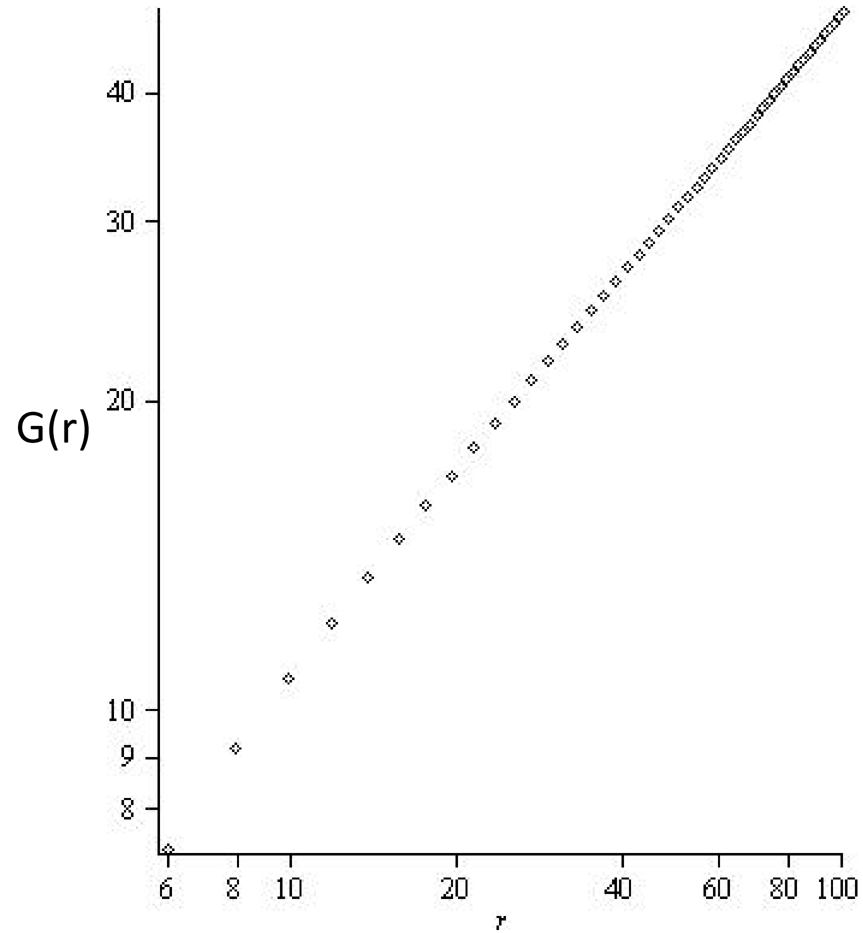
Critical condition for trapped surface formation

$$\frac{4\pi^2}{N_c^2}\mu(z_1 z_2)^{3/2} \geq \sqrt{G(r)}$$

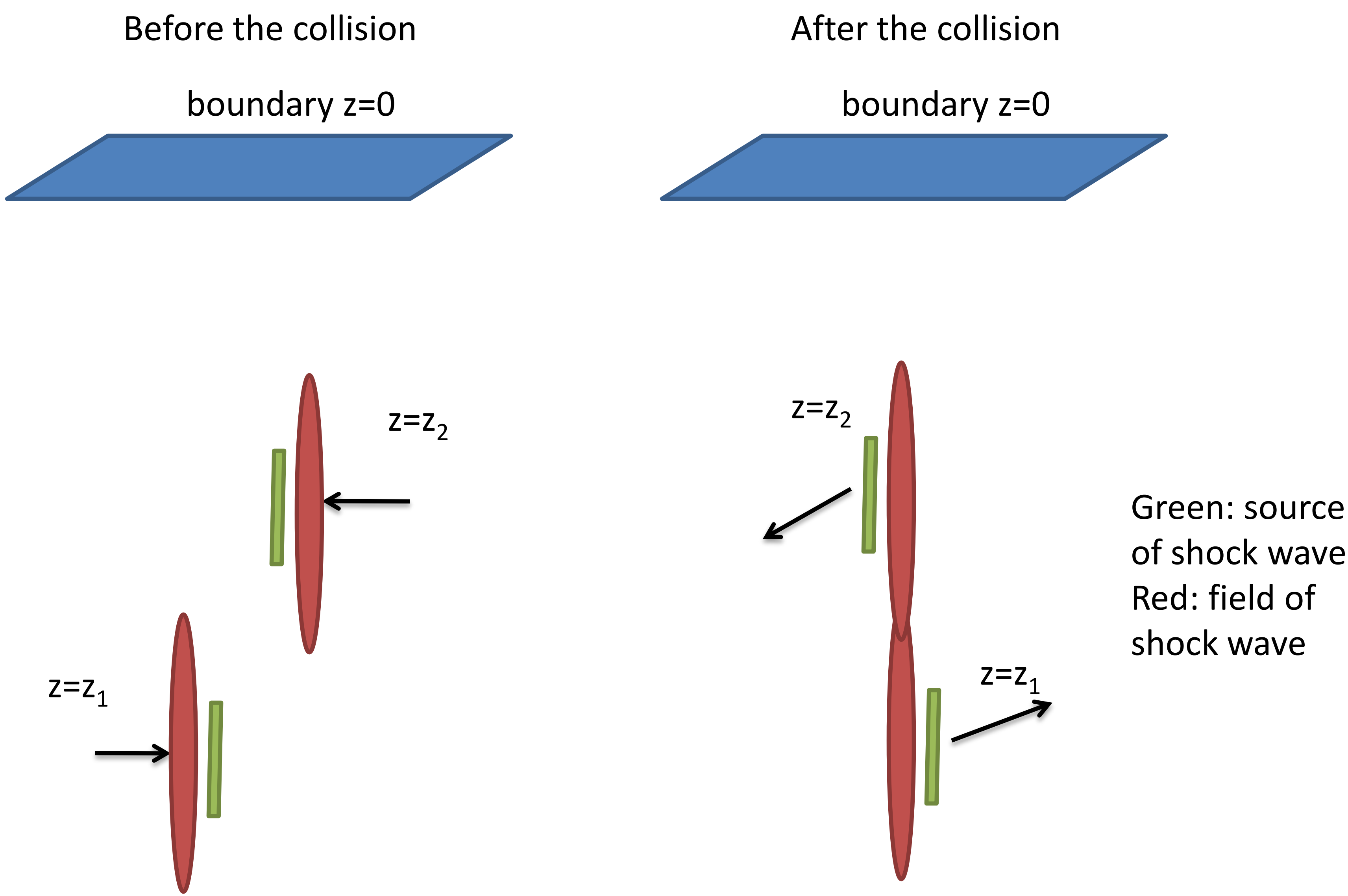
$$\text{with } r = \frac{(z_1^4 + z_2^4)^2}{(z_1 z_2)^4}$$

In the limit z1/z2>>1,

$$\sqrt{G(r)} \approx (\frac{z_1}{z_2})^{3/2}$$



NLO stress tensor



Stress tensor at NLO

LO $$T_{++} = \mu_1 \delta(x^+), \quad T_{--} = \mu_2 \delta(x^-)$$

NLO $$T_{++} = 16\pi G_5 \mu_1 \mu_2 [-x^{+2} \theta(z_2 - \tau) + \frac{x^- z_2^3}{2x^+} \delta(z_2 - \tau)]$$

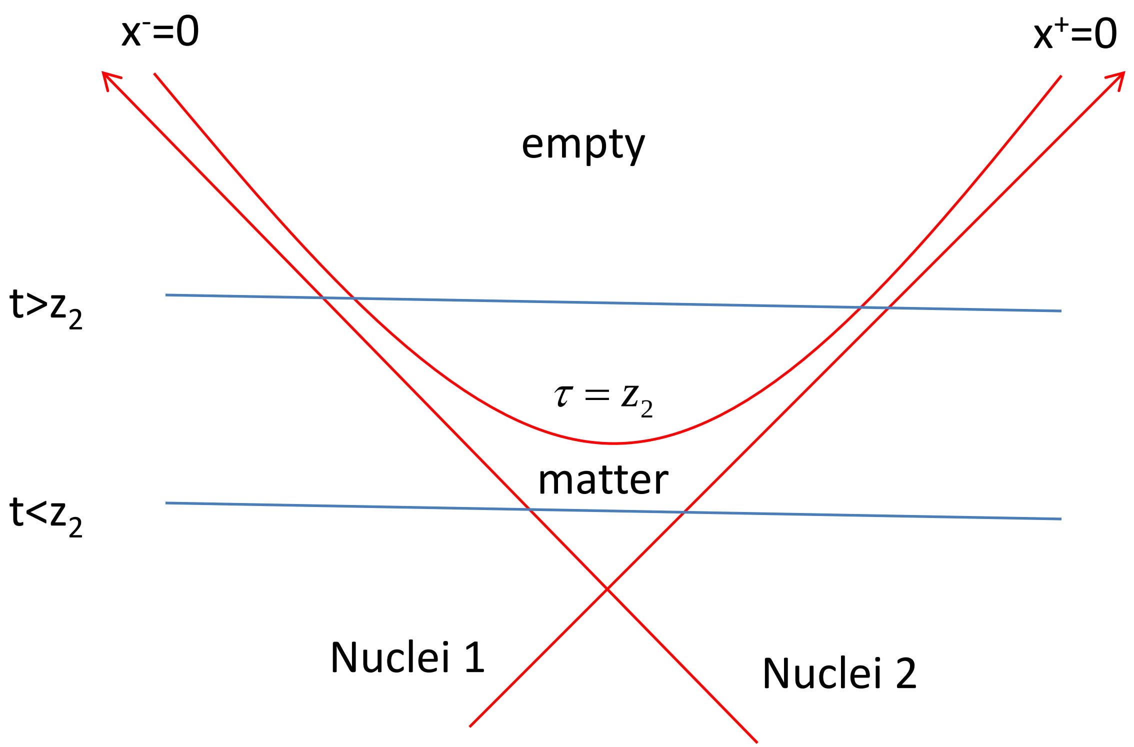
$$T_{--} = 16\pi G_5 \mu_1 \mu_2 [-x^{-2} \theta(z_2 - \tau) + \frac{x^+ z_2^3}{2x^-} \delta(z_2 - \tau)]$$

$$T_{+-} = \frac{1}{2} T_{\perp\perp} = 16\pi G_5 \mu_1 \mu_2 [2\tau^2 \theta(z_2 - \tau) - \frac{z_2^3}{2} \delta(z_2 - \tau)]$$

$z_1 > z_2$

In the limit $z_2 \to \infty$
Grumiller and Romatschke 08
Kovchegov et al 08,09

Condition for equilibration



Perturbation has to break down in order for matter not to fly apart, otherwise equilibration is not possible

Estimate of critical impact parameter

When the source is significantly deviated from its original position in radial direction, the perturbation should break down

$$t \sim \frac{\# z_2}{u^z} = \frac{\# z_2}{32\pi G_5 \mu z_2^3} \le z_2$$

Critical energy density for matter equilibration

$$\frac{16\pi^2}{N_c^2}\mu(z_1 z_2)^{3/2} \ge \#(\frac{z_1}{z_2})^{3/2}$$

In agreement with trapped surface analysis when z1/z2>>1

Conclusion

- The exsitence of critical impact parameter is shown, as analogous to flat space. The entropy associated with the trapped surface shows a first order transition in impact parameter.
- The limitation of point shock wave in modeling HIC is commented. The simple wall-on wall collision is proposed. There also exists a lower bound for energy, in the absence of impact parameter.
- The origin of the critical condition in matter equilibration in wall shock waves collision is understood in terms of NLO stress tensor.

Thank you!