N. Evans, T. Kalaydzhyan, K.-y. Kim,
I.K., [hep-th, Oct.'10]

Chiral symmetry breaking in an expanding plasma

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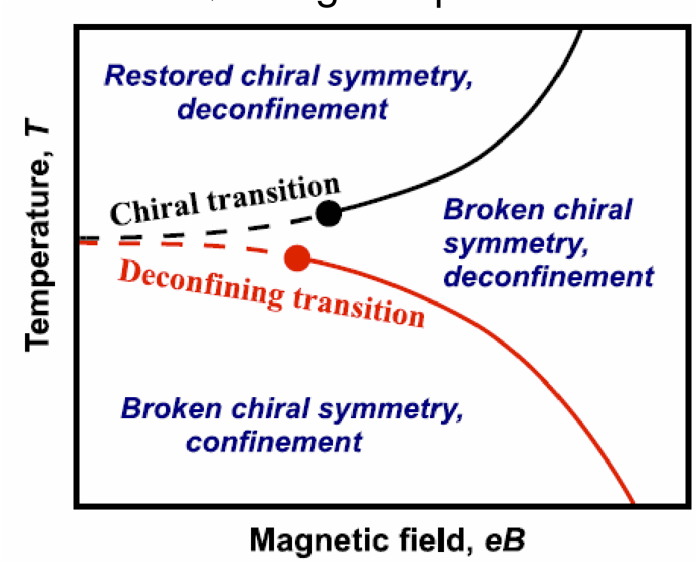
Motivation

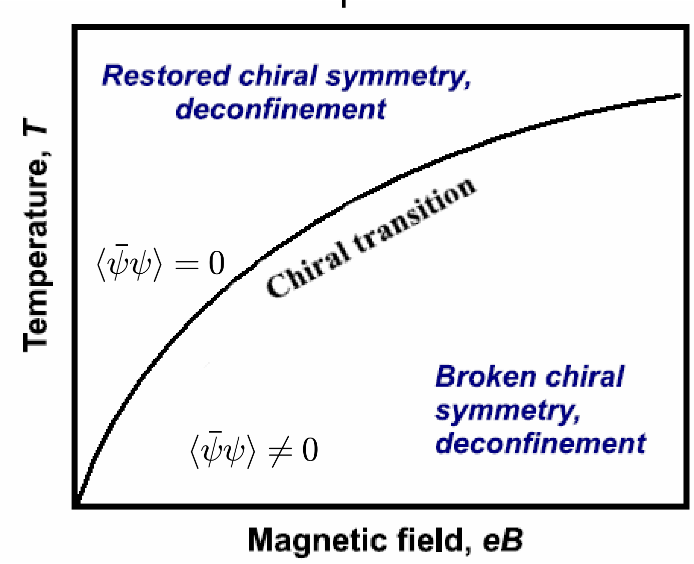
Very strong magnetic fields *B* expected in QGP: at first moments (τ ~ 1 fm/c) of Au-Au collision at RHIC the strength of the magnetic field may reach *eB* ~ (10-100 MeV)^2

B-T phase diagrams (sketch):

Quark-gluon plasma



N=4 plasma



B>0:

- QGP: splitting of the confinement and chiral transitions (Mizher et al., '10)
- N=4: only chiral transition

Motivation

GOAL: Real-time time-dynamics of an N=4 plasma in the presence of a magnetic field

Overview:

• use AdS/CFT

• take Janik's boost-invariant background+additional B-field as a dual of an expanding N=4 plasma with B-field

• embed branes into this background to add fundamental fields ("quarks") to the plasma

• determine the quark condensate c(τ,B) as a function of time and B

=> order parameter for the chiral transition

Janik's proposal for a boost-invariant background

N=4 plasma expansion dual to AdS black hole with horizon $r_H \sim 1/\tau^{1/3}$

Time-dependent type IIB sugra background:

$$\frac{ds^2}{R^2} = \frac{1}{z^2}\left(-e^{a(\tau,z)}d\tau^2 + e^{b(\tau,z)}\tau^2 dy^2 + e^{c(\tau,z)}dx_\perp^2\right) + \frac{dz^2}{z^2} + d\Omega_5^2$$

At late times, Janik introduces scaling variable $v = z/\tau^{1/3}$

$$a(\tau,z) = a_0(v) + a_1(v)\tau^{-2/3} + \cdots, etc.$$

and solves Einstein equations order by order (regularity)

$$a(\tau,z) = \ln\left(\frac{(1-v^4/3)^2}{1+v^4/3}\right) + 2\eta_0\frac{(9+v^4)v^4}{9-v^8}\frac{1}{\tau^{2/3}} + \cdots \equiv \varepsilon(\tau)z^4 + \cdots$$

with

$$\varepsilon(\tau) = \frac{1}{\tau^{4/3}} - \frac{2\eta_0}{\tau^2}$$

energy density of a boost-invariant expanding viscous plasma

Flavour in N=4 plasma

Time-dependent D7-brane embedddings in N=4 plasma are described by

$$S_{DBI} = -T_{D7} \int d^8\xi \sqrt{-\det(P[G]_{ab} + 2\pi\alpha' F_{ab})}$$

with constant magnetic field

$$F_{12} = B/(2\pi\alpha')$$

Embedding Lagrangian for $L(\tau, \rho)$:

$$\mathcal{L}_{DBI} = \mathbb{A}\sqrt{\left(1 + \mathbb{C}\frac{B^2}{(\rho^2 + L^2)^2}\right)\left(1 + L'^2 - \mathbb{B}\frac{\dot{L}^2}{(\rho^2 + L^2)^2}\right)}$$

Janik's background enters $\mathbb{A}, \mathbb{B}, \mathbb{C}$ and $1/z^2 = r^2 = \rho^2 + L^2 \quad (R = 1)$

e.o.m. for embedding = partial differential equation (PDE)

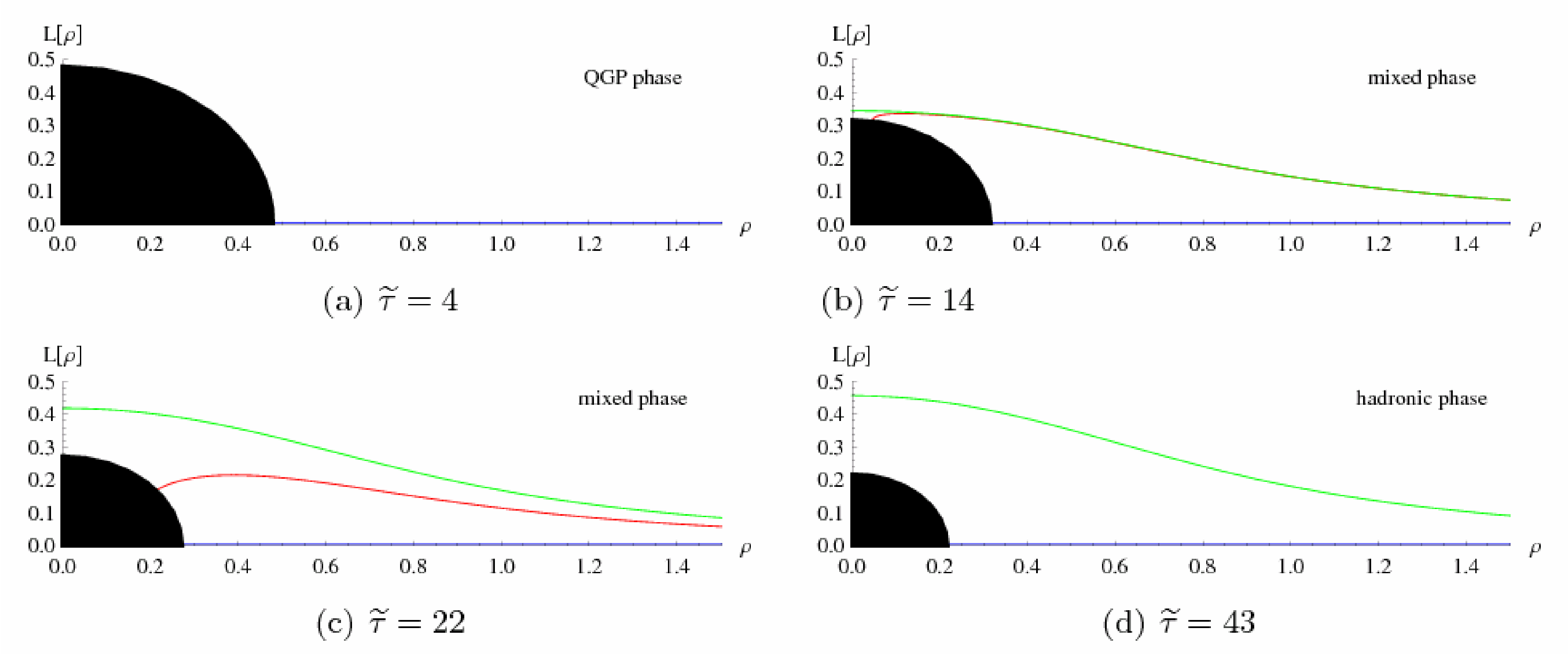
Quasi-equilibrium approach

ansatz:

$$L(\tau,\rho) = f_0(\rho) + \sum_{i=1}^{\infty} f_i(\rho)\tau^{-\frac{i}{3}}$$

two forces: - black hole (attractive)
- B-field (repelling)

embedding profiles:

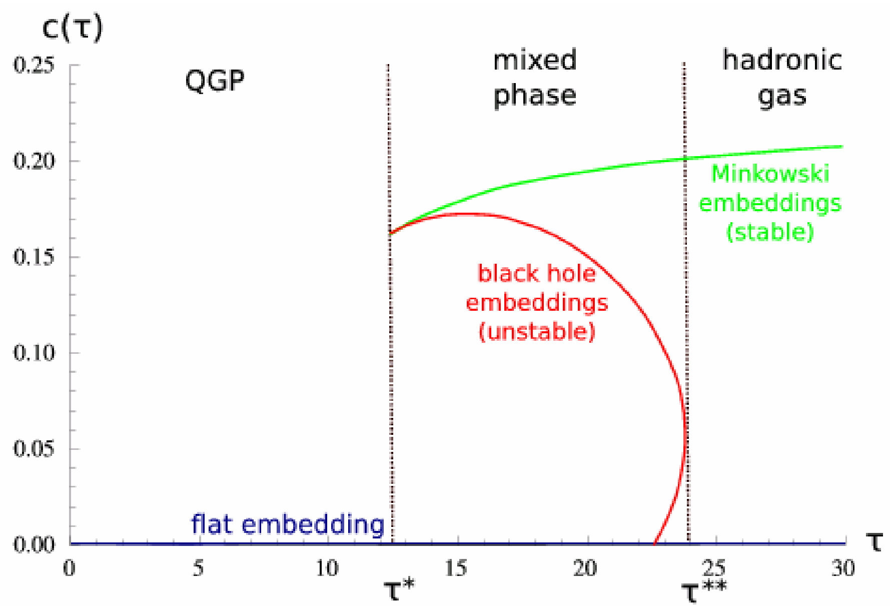


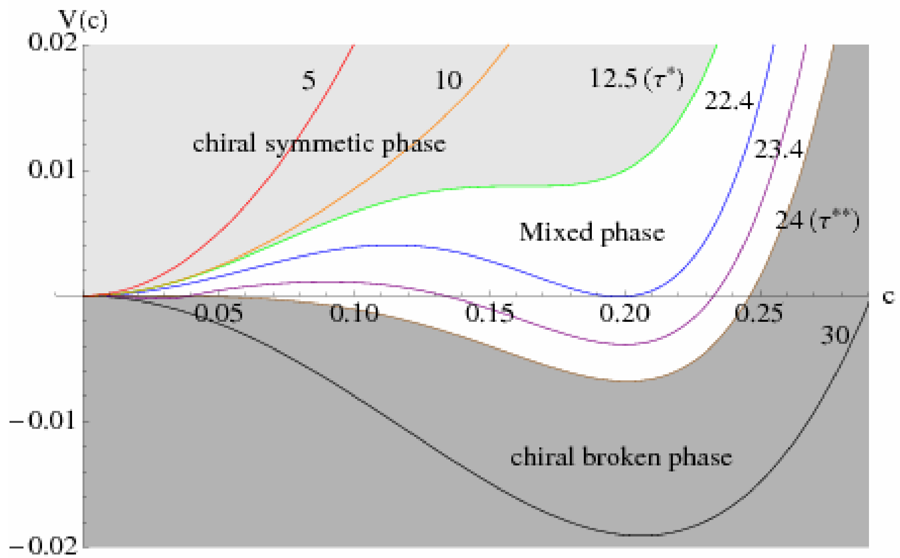
Quasi-equilibrium condensate c(τ) and potential V(c)

order parameter $c(\tau) = -\langle\bar{\psi}\psi\rangle$ from asympt. embedding behaviour:

$$L(\tau,\rho) \sim \frac{c(\tau)}{\rho^2}$$

evolution of extremum states:





PDE approach: real-time dynamics

- quasi-equilibrium approach determines the potential V(c)
- but it reflects the actual real-time dynamics of the states of the system only at late times, when the system is close to equilibrium
- breaks down near the time of the chiral transition, when the system is far away from equilibrium

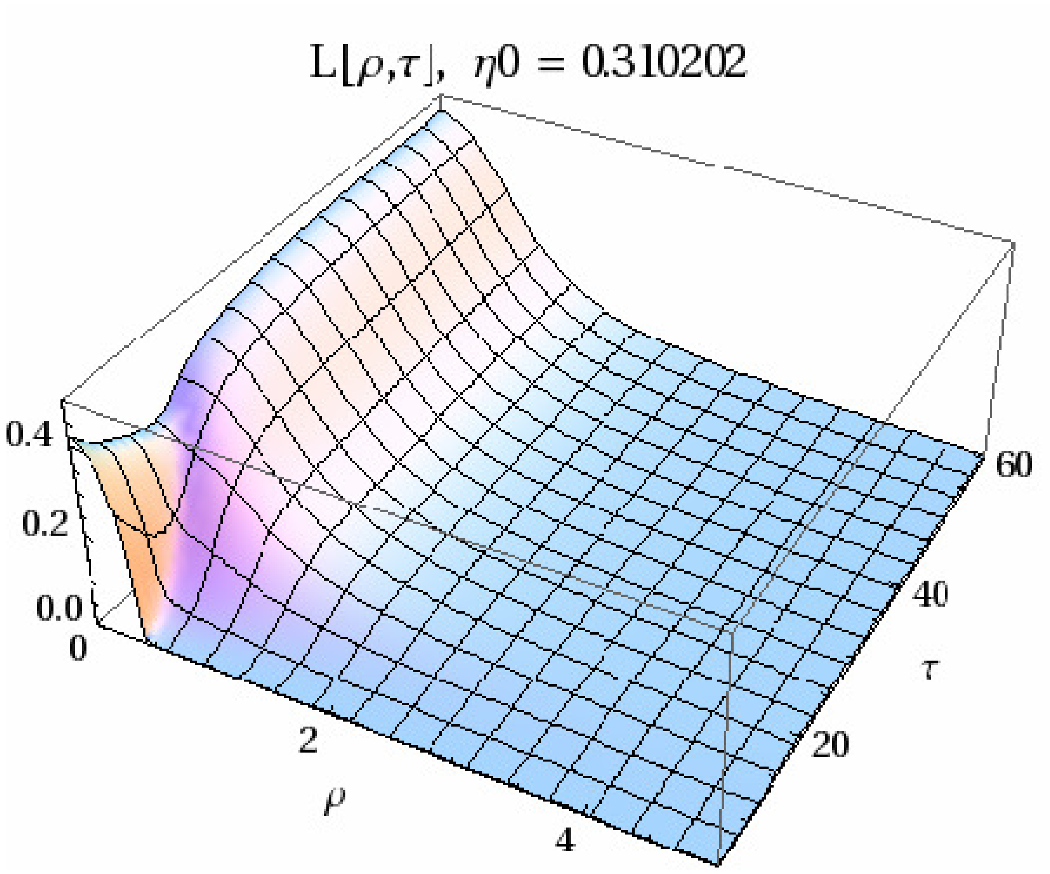
=> need to solve PDE directly

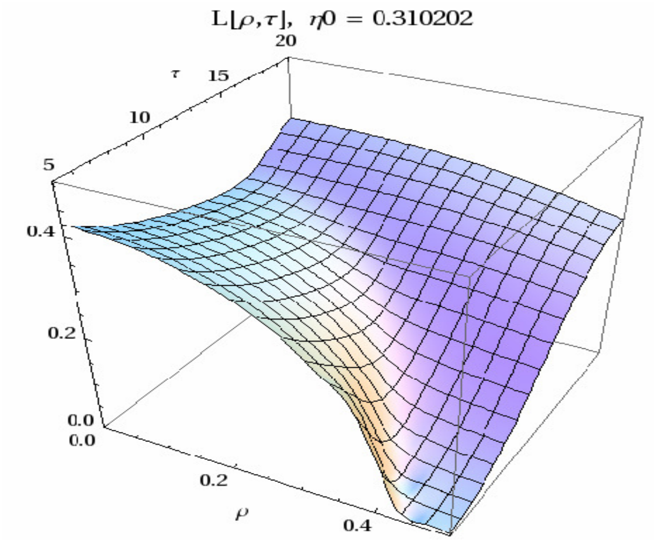
- initial and boundary conditions

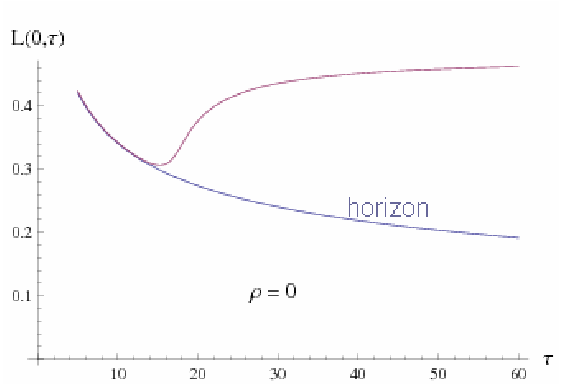
$$L(\tau \to \infty, \rho) = f_0(\rho) \ , \qquad \partial_\tau L(\tau \to \infty, \rho) = 0$$
$$\partial_\rho L(\tau, \rho = 0) = 0 \ , \qquad L(\tau, \rho \to \infty) = 0$$

- solve PDE in reverse time order

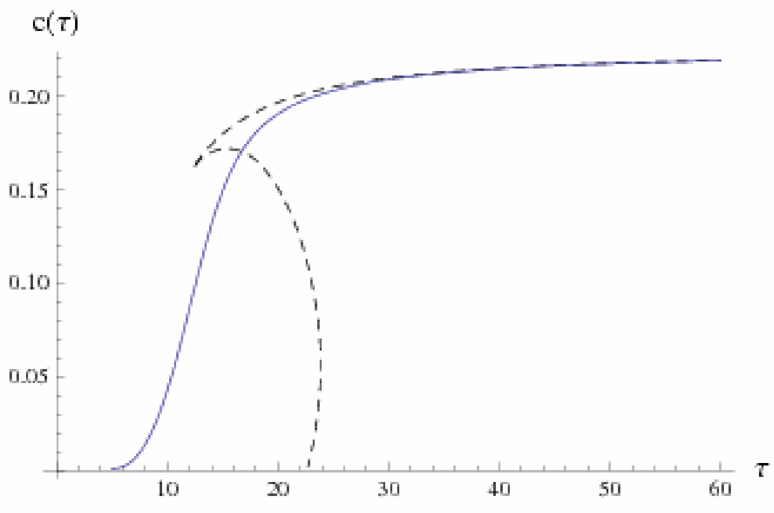
Numerical embedding profile L(ρ,τ)



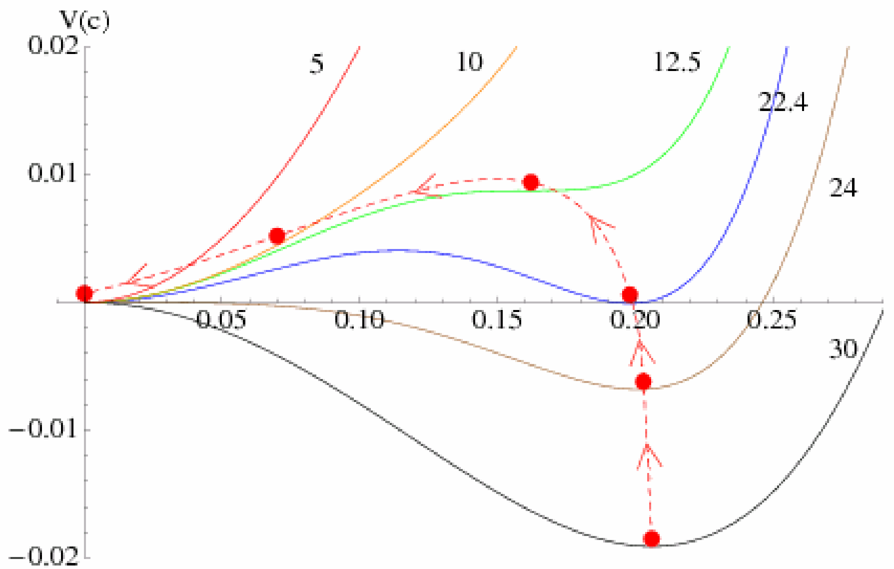




Condensate c(τ) and potential V(c)



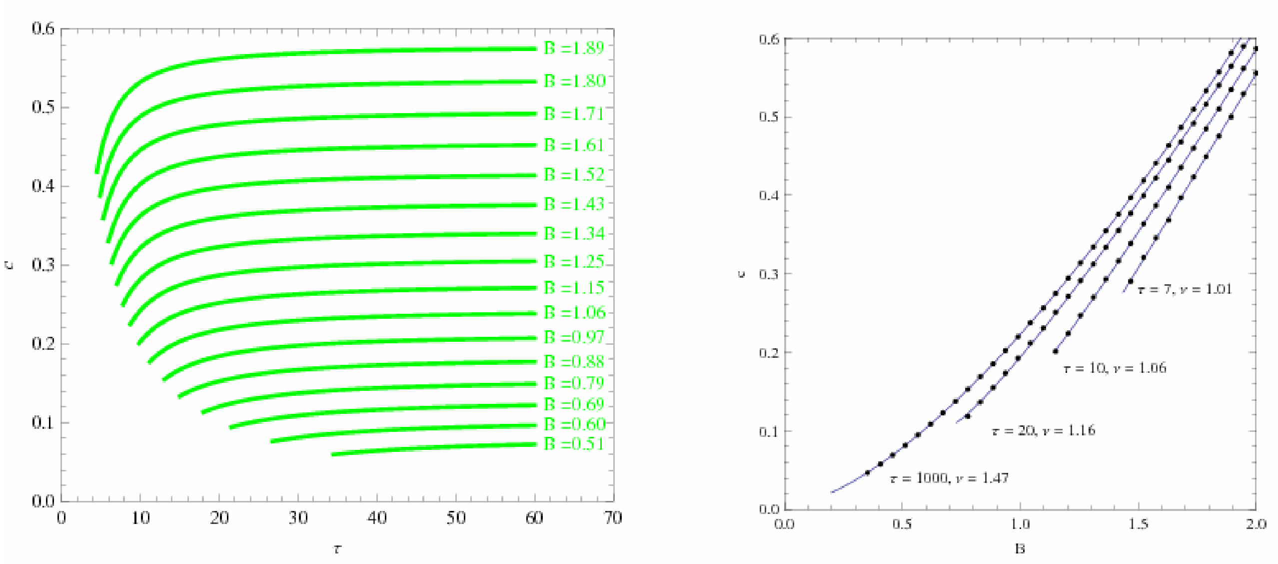
Condensate $\tilde{c}(\tilde{\tau})$ (blue solid curve) compared with the "quasi-equilibrium" condensate (dashed curve).



Time evolution of condensate.

The path of the red dots in the effective potential corresponds the evolution of the embedding profile.

Condensate *c(τ,B)* – dependence on the magnetic field



(a) condensate $c(\tau)$ for various B (b) $c(B) \propto B^{\nu}$ for $\tau = 7, 10, 20, 1000$

- *c* increases with *B*:

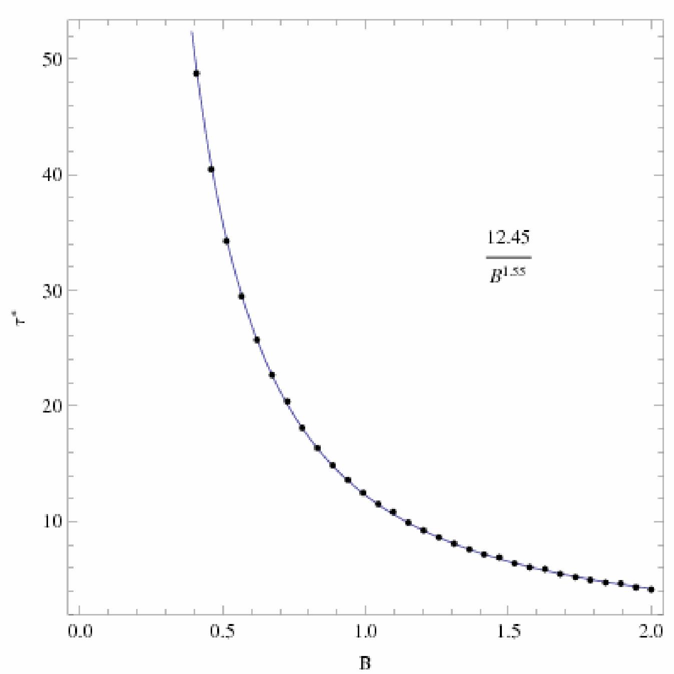
$$c \sim B^{\nu} \quad \begin{cases} \nu \approx 1 & \text{(high } T) \\ \nu \approx 1.5 & \text{(low } T) \end{cases}$$

(B sufficiently strong)

in qualitative agreement with other studies:

- NJL: Klevansky and Lemmer ’89 (ν=2):
- chiral perturbation theory Shushpanov, Smilga ’97 (strong B: ν=3/2, weak: ν=1)
- lattice: Buividovich et al. ’08 (SU(2), ν=1), Kalaydzhyan et al (SU(3), to be published)
- AdS/CFT: Zayakin ‘08 (confining, large N, ν=2)

Time of the phase transition



τ_* as a function of B $(\varepsilon_0 = 1)$

$$\tau_* = 12.45 B^{-1.55}$$

0.05 correction due to $\eta \neq 0$

- the higher B the earlier the chiral transition (transition is shifted to higher temperatures)
- increase of transition temperature in qualitative agreement with Mizher, Chernodub, Fraga, ‘10
- in the adiabatic approximation (no viscosity), we get $T_c \sim B^{1/2}$ $(T \sim \tau^{-1/3})$

  in agreement with Shushpanov, Smilga ’97

Conclusions

Progress in describing the real-time evolution of an N=4 expanding plasma with fundamental matter using AdS/CFT: In particular, we studied the chiral phase transition by embedding D7-branes into Janik's time-dependent AdS black hole background (plus B-field).

Results:

1. AdS/CFT description of the real-time dynamics of this non-equilibrium process.

2. We computed the quark condensate as a function of time and B.

3. Interesting phenomenology:
   - $c(\tau, B) = -\langle\bar{\psi}\psi\rangle$ increases with B^{ν} with $1<\nu<3/2$
   - critical time/temp. $\tau_* \sim B^{-1.55}$