

# $J/\psi$ Production with NRQCD

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# Production and Decay Rates of Heavy Quarkonia

**Heavy quarkonia:** Bound states of heavy quark and its antiquark.

- Charmonia ( $c\bar{c}$ ) and Bottomonia ( $b\bar{b}$ )
- Top decays too fast for bound state

**The classic approach: Color-singlet model**

- Calculate cross section for heavy quark pair in physical **color-singlet** (= color neutral) state. In case of  $J/\psi$ :  $c\bar{c}[{}^3S_1^{[1]}]$
- Multiply by quarkonium wave function (or its derivative) at origin
- Strong disagreement with Tevatron data

**Nonrelativistic QCD (NRQCD):**

- 1995: Rigorous effective field theory by Bodwin, Braaten, Lepage
- Based on **factorization of soft and hard scales**  
(Scale hierarchy:  $Mv^2, Mv \ll \Lambda_{\text{QCD}} \ll M$ )
- Could explain hadroproduction at Tevatron

# $J/\psi$ Production with NRQCD

**Factorization theorem:**  $\sigma_{J/\psi} = \sum_n \sigma_{c\bar{c}[n]} \cdot \langle O^{J/\psi}[n] \rangle$

- $n$ : Every possible Fock state, including **color-octet** states.
- $\sigma_{c\bar{c}[n]}$ : Production rate of  $c\bar{c}[n]$ , calculated in perturbative QCD.
- $\langle O^{J/\psi}[n] \rangle$ : Long distance matrix elements (LDMEs): describe  $c\bar{c}[n] \rightarrow J/\psi$ , universal, extracted from experiment.

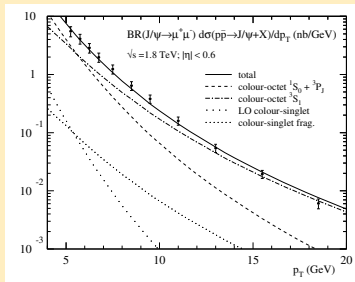
**Scaling rules:** MEs scale with relative velocity  $v$  ( $v^2 \approx 0.2$ ):

scaling	$v^3$	$v^7$	$v^{11}$
$n$	$^3S_1^{[1]}$	$^1S_0^{[8]}, ^3S_1^{[8]}, ^3P_{0/1/2}^{[8]}$	...

- **Double expansion** in  $v$  and  $\alpha_s$ .
- Leading term in  $v$  ( $n = ^3S_1^{[1]}$ ) equals **color-singlet model**.

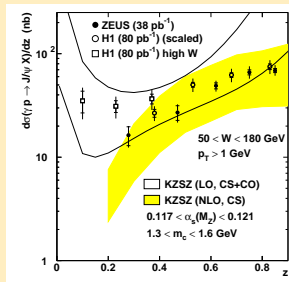
# Production of $J/\psi$ : NRQCD vs. Experiment (History)

## Hadroproduction at Tevatron:



- Color octet states important  
 $\Rightarrow$  **Great success** for NRQCD

## Photoproduction at HERA:



- MEs from fits to Tevatron data.
- Importance of color octet **unclear**

**This work:** **NLO** NRQCD calculation for photo- and hadroproduction  
 $\Rightarrow$  Aim: Establish universality of long distance matrix elements.

# Production of $J/\psi$ : Summary of Calculations

## Hadroproduction:

	$3S_1^{[1]}$	$1S_0^{[8]}, 3S_1^{[8]}, 3P_{0/1/2}^{[8]}$
<b>Born</b>	Baier, Rückl (1980)	Cacciari, Krämer (1996)
<b>NLO</b>	Campbell et al. (2007)	<b>M.B., Kniehl (2010)</b>

## Photoproduction:

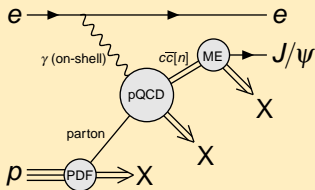
	$3S_1^{[1]}$	$1S_0^{[8]}, 3S_1^{[8]}, 3P_{0/1/2}^{[8]}$
<b>Born</b>	Berger, Jones (1981)	Ko, Lee, Song (1996)
<b>NLO</b>	Krämer (1995)	<b>M.B., Kniehl (2009)</b>

## Open question of ME universality:

- NLO NRQCD calculation: Only after **13 years!**
- Difficulty: Virtual corrections to  **$P$  states**

# Direct $J/\psi$ Production

**Factorization formulas:** (e.g. photoproduction)



- Convolute partonic cross sections with **proton PDFs**:

$$\sigma_{\text{hadr}} = \sum_i \int dx f_{i/p}(x) \cdot \sigma_{\text{part},i}$$

- **NRQCD factorization:**

$$\sigma_{\text{part},i} = \sum_n \sigma(\gamma i \rightarrow c\bar{c}[n] + X) \cdot \langle O^{J/\psi}[n] \rangle$$

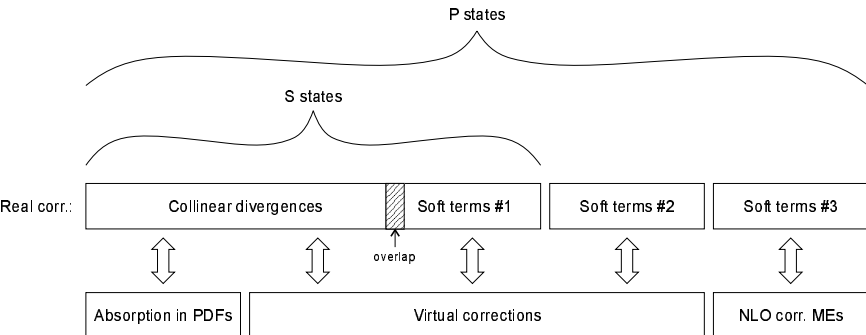
**Amplitudes for  $c\bar{c}[n]$  production by projector application, e.g.:**

$$A_{c\bar{c}[3S_1^{[1/8]}]} = \varepsilon_\alpha \text{Tr} [C \Pi^\alpha A_{c\bar{c}}] |_{q=0}$$

$$A_{c\bar{c}[3P_J^{[8]}]} = \varepsilon_{\alpha\beta} \frac{d}{dq_\beta} \text{Tr} [C \Pi^\alpha A_{c\bar{c}}] |_{q=0}$$

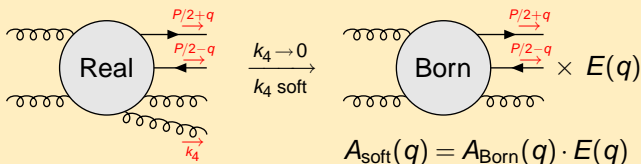
- $A_{c\bar{c}}$ : Amputated pQCD amplitude for open  $c\bar{c}$  production.
- $q$ : Relative momentum between  $c$  and  $\bar{c}$ .

# Overview of IR Singularity Structure



# Structure of Soft Singularities

## Soft limits of the real corrections:



## S and P states: Soft #1 + Soft #2 + Soft #3 terms:

$$A_{\text{soft},s} = A_{\text{soft}}(0) = A_{\text{Born},s} \cdot E(0)$$

$$A_{\text{soft},p} = A'_{\text{soft}}(0) = A_{\text{Born},p} \cdot E(0) + A_{\text{Born},s} \cdot E'(0)$$

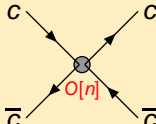
$$|A_{\text{soft},s}|^2 = |A_{\text{Born},s}|^2 \cdot E(0)^2$$

$$|A_{\text{soft},p}|^2 = |A_{\text{Born},p}|^2 \cdot E(0)^2 + 2 \operatorname{Re} A_{\text{Born},s}^* A_{\text{Born},p} \cdot E(0) E'(0) + |A_{\text{Born},s}|^2 \cdot E'(0)^2$$



# Radiative Corrections to Long Distance MEs

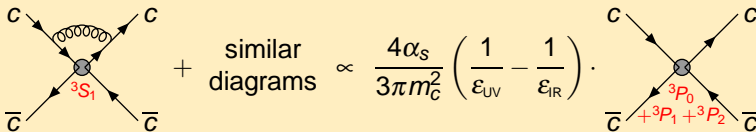
In NRQCD: Long distance MEs =  $c\bar{c}$  scattering amplitudes:

$$\langle O^{J/\psi}[n] \rangle =$$


$O[n]$  = 4-fermion operators

$$(n = {}^3S_1^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_{0/1/2}^{[8]}, \dots)$$

Corrections to  $\langle O^{J/\psi}[{}^3S_1^{[1/8]}] \rangle$  with NRQCD Feynman rules:

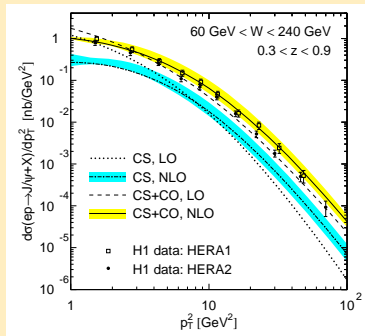
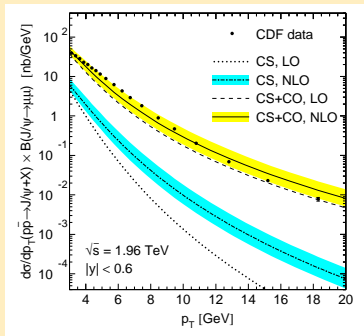


$$+ \text{similar diagrams} \propto \frac{4\alpha_s}{3\pi m_c^2} \left( \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) \cdot$$

- UV singularity cancelled by renormalization of 4-fermion operat.
- IR singularity cancels soft #3 terms of  $p$  states.

# Combined Fit to Tevatron and HERA (1)

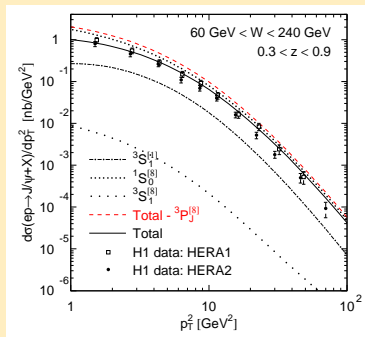
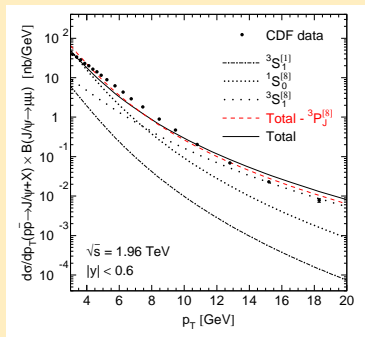
## Fit CO LDMEs to transverse momentum distributions:



- Best fit values:  $\langle O^{J/\psi}[^1S_0^{[8]}] \rangle = 0.033 \text{ GeV}^3$ ,  
 $\langle O^{J/\psi}[^3S_1^{[8]}] \rangle = 0.0043 \text{ GeV}^3$ ,  $\langle O^{J/\psi}[^3P_0^{[8]}] \rangle = -0.0072 \text{ GeV}^5$
- Fit very sensitive to input parameters: “Underconstrained”

# Combined Fit to Tevatron and HERA (2)

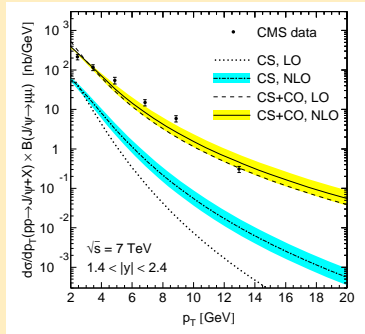
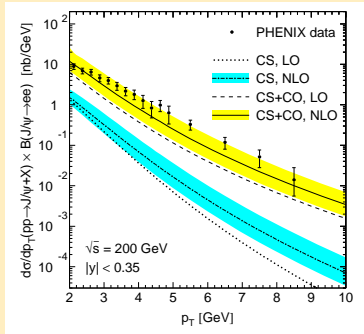
## Contribution of individual states:



- Hadroprod.: Short-distance  $\sigma(c\bar{c}[{}^3P_J^{[8]}])$  **negative** for  $p_T \gtrsim 7$  GeV
- But: Short-distance cross sections and LDMEs **unphysical** (NRQCD-scale dependence)  $\implies$  No problem!

# Predictions for RHIC and LHC

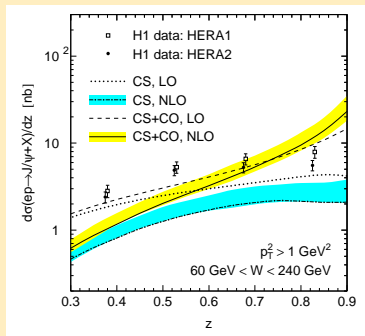
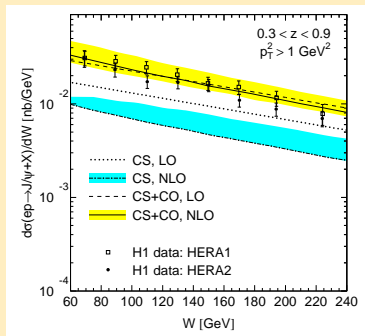
## Use LDMEs to make RHIC and LHC predictions:



- Also RHIC and LHC data **well described** by CS+CO.
- Like at Tevatron: **CS** orders of magnitudes **below** the data.
- These data **not part** of the fit, outcome not trivial.

# Further Predictions for HERA

## Use LDMEs to make $W$ and $z$ distribution predictions:



- Proton rest frame:  $z =$  Fraction of photon energy going to  $J/\psi$ .
- $z \lesssim 0.45$ : **Resolved** photoproduction important (not yet included).
- $W$  distribution also well described. Data not part of the fit.

# Summary (1)

## Our project: Test NRQCD

- NRQCD provides rigorous **factorization theorem** for production and decay of heavy quarkonia: Include **color-octet** (CO) states.
- But: Need to proof **universality** of CO LDMEs.
- **This work**: Technological breakthrough: After **13 years** finally NLO NRQCD photo- and hadroproduction calculation.

## Our Results:

- **CSM predictions**: Could verify all previous results: CS contributions **far below data** in all considered experiments.
- **Fitted** CO LDMEs to  $p_T$  distributions at Tevatron and HERA.
- Used LDMEs for RHIC, LHC and HERA  $W$  and  $z$  distributions  $\implies$  CS+CO: Good **agreement** with data.

# Summary (2)

## Discussion of Fit:

- **NLO** hadroproduction  $p$  states: Shape changes, even negative.  
⇒ Fit **all three** CO LDMEs (not linear combination like at LO).
- Negative unphysical quantities no problem.
- Photo- and hadroproduction **consistently described** by NRQCD.

## Still to be done:

- Include **resolved** photoproduction.
- Extend analysis to  $e^+e^-$  collisions (LEP,  $B$  factories).
- Include **feeddown** processes.
- Do **polarization** analysis.