



On BCFW shifts of integrands and integrals

(based on [arXiv:1008.3101](https://arxiv.org/abs/1008.3101) [hep-th])

Rutger Boels
University of Hamburg



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scattering amplitudes are interesting ...



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question:

Is there a recursive solution to the Yang-Mills S-matrix to all loop orders?



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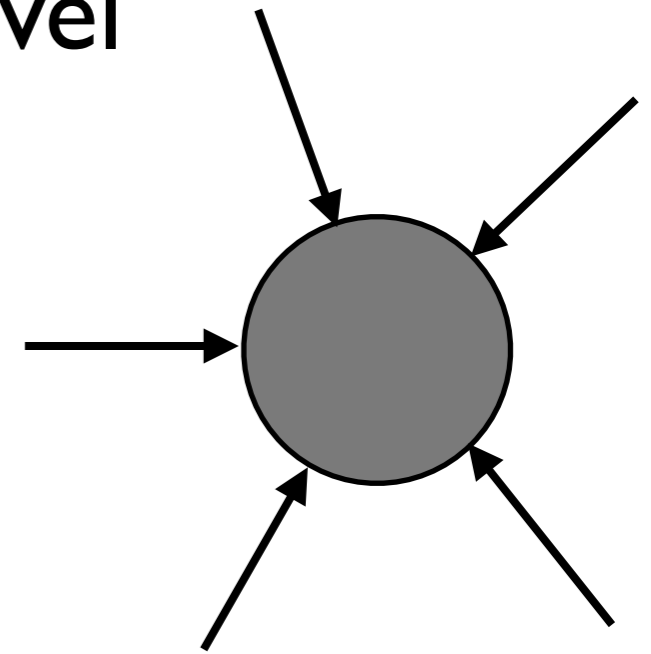
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this talk: **maybe...**



BCFW on-shell recursion at tree level

scattering amplitudes are functions with physical singularities: poles and branch cuts

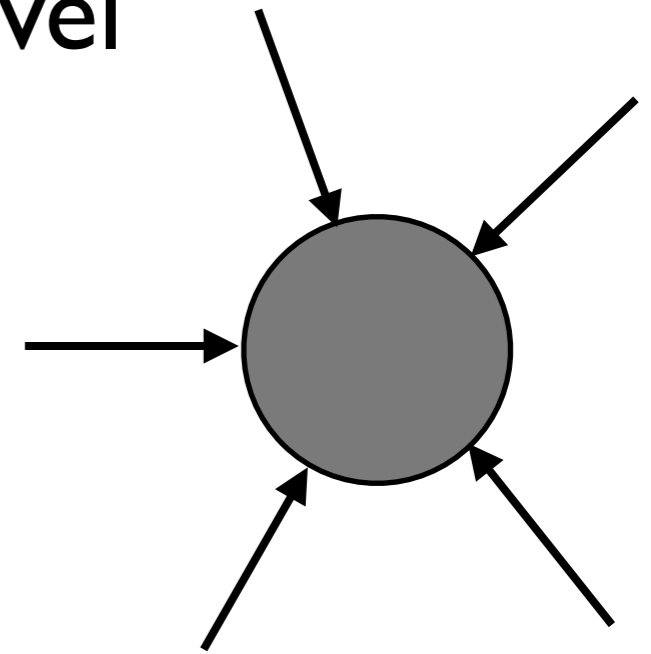




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- no obvious single complex variable

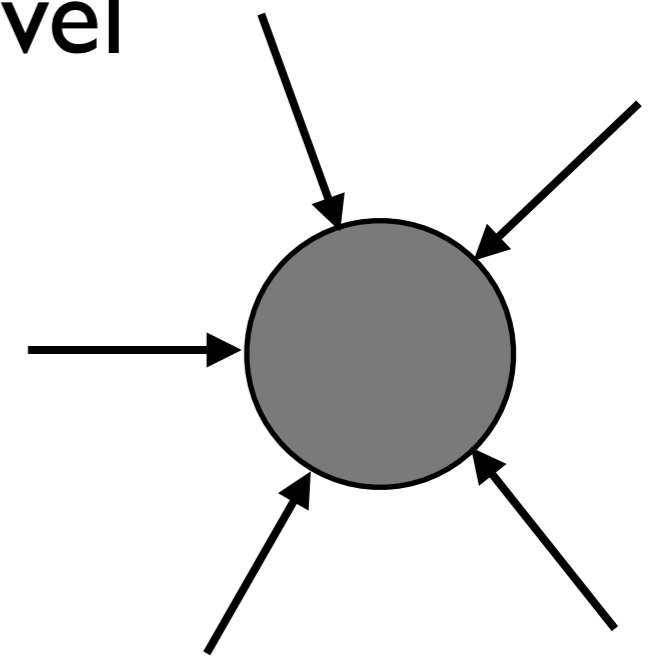




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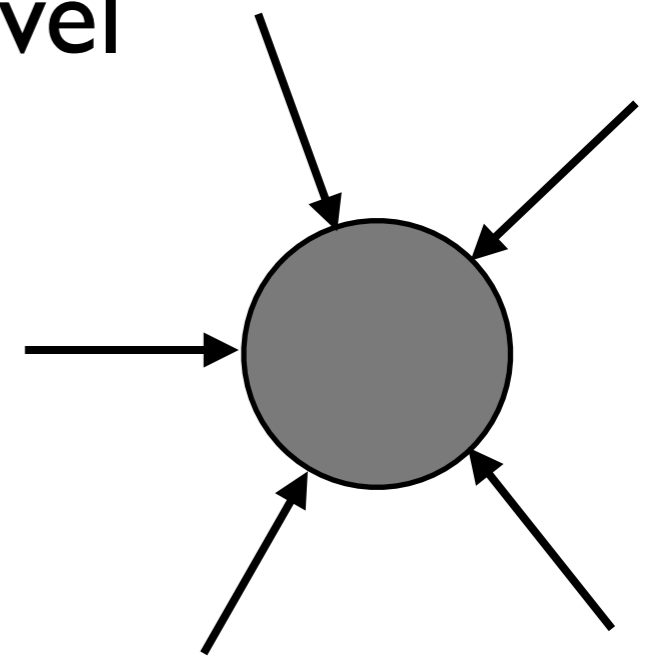
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Pick any two legs

$$k_1 \rightarrow k_1 + zq$$

$$k_2 \rightarrow k_2 - zq$$



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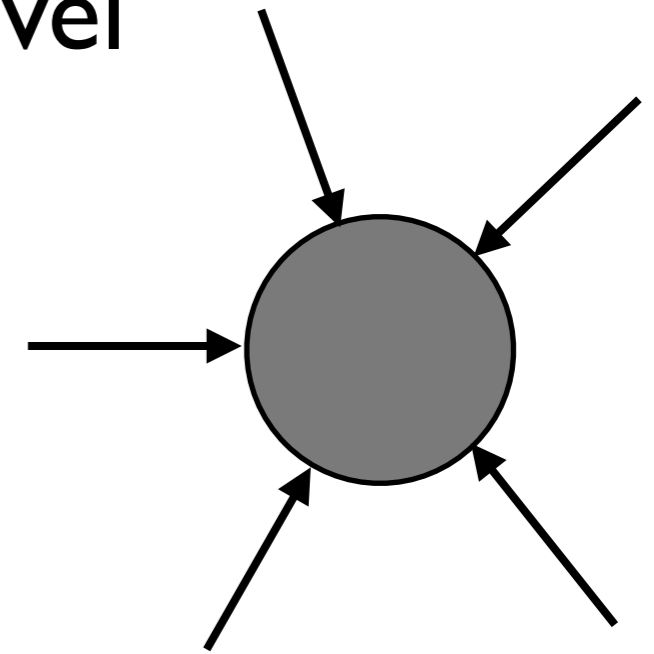




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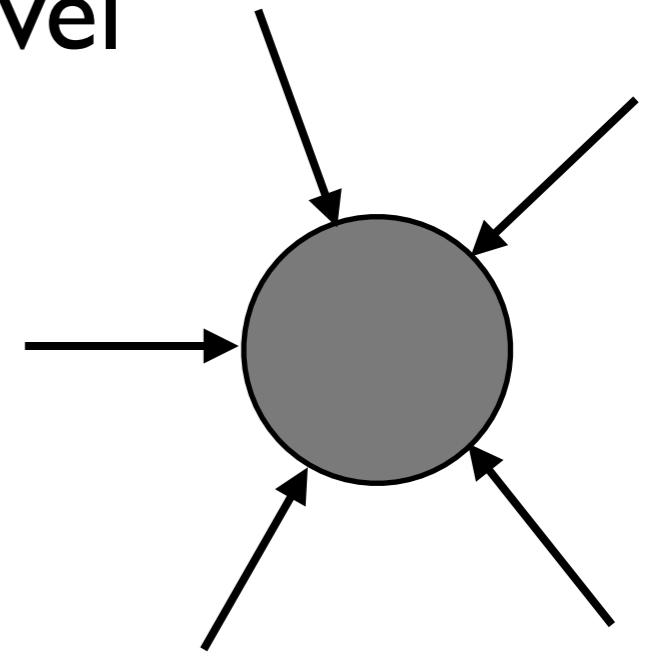
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mass of particles unchanged



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- solution for complex q
- amplitude $A(z)$, want $A(0)$
- in general $A(z)$ has poles and branch cuts in the z -plane

mass of particles unchanged



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finite z residues products of lower point amplitudes **@ tree level**

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- study $A(\infty)$: probes UV, related to Regge behavior
- residues absent in YM, gravity (e.g. [Arkani-Hamed et al, 08]) and strings (e.g. [Cheung, et.al. 10], [Boels et.al., 10])
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- what about loops?



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see next slide...



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- BCFW shift involves a special light-cone vector q : use as gauge choice [Arkani-Hamed, Kaplan, 08]
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- explicit results at one loop in background field setup, seems to reproduce structure of rational parts ([Bern et.al, 05-06])
 - only deviations from tree pattern in non-susy theories?



Discussion and conclusions

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